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A generalized *AZ*-non-Maxwellian velocity distribution function for space plasmas

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A more generalized form of the non-Maxwellian distribution function, i.e., the *AZ*-distribution function is presented. Its fundamental properties are numerically observed by the variation of three parameters: α (rate of energetic particles on the shoulder), r (energetic particles on a broad shoulder), and q (superthermality on the tail of the velocity distribution curve of the plasma species). It has been observed that (i) the *AZ*- distribution function reduces to the (r, q)- distribution for $\alpha \to 0$; (ii) the *AZ*- distribution function reduces to the q- distribution for $\alpha \to 0$, and $r \to 0$; (iii) the *AZ*-distribution reduces to Cairns-distribution function for $r \to 0$, and $q \to \infty$; (iv) the *AZ*-distribution reduces to Vasyliunas Cairns distribution for $r \to 0$, and $q = \kappa + 1$; (v) the *AZ*-distribution reduces to kappa distribution for $\alpha \to 0$, $r \to 0$, and $q = \kappa + 1$; and (vi) finally, the *AZ*-distribution reduces to Maxwellian distribution for $\alpha \to 0, r \to 0$, and $q \to \infty$. The uses of this more generalized *AZ*-distribution function in various space plasmas are briefly discussed. *Published by AIP Publishing.*
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I. INTRODUCTION

In the plasma, the constituent particles (viz., electrons, negative/positive ions, etc.) collide with one another constantly and move with different velocities. The plasma particles may interchange their kinetic energy (K.E) and momentum (P) due to these interactions to attain thermal equilibrium. The distribution commonly used to describe the plasma particles is the Maxwellian distribution[1–3] that depends exponentially on the particle velocities. The Maxwellian distribution is best applicable to the plasma systems where the particles are treated in thermal equilibrium. As far as the deviation from Maxwellian to non-Maxwellian is concerned, the long-range interactions[4] (like Coulomb and gravitational) may be considered a reasonable source for this.

There are many space plasma phenomena, e.g., interstellar medium, thermoshpere ionosphere, solar wind, planetary magnetosphere, and magnetosheaths, where the non-Maxwellian velocity distribution function is very common.[5–8] On the basis of this point, Vasyliunas[5] was the first to analyze the electrons energy spectra within the sheet of plasma. He introduced a function to model the velocity distribution of high energy electrons within the plasmas called kappa distribution. In 1995, observations were made by the Freja satellite[9] and Viking spacecraft[10] to study the rarefaction of ion number density (cavitons). Cairns *et al.*[6] have presented another sort of non-thermal distribution known as Cairns distribution. They investigated the existence of cavitons in the plasma systems where the particles are not in thermal equilibrium, as studied by the Freja satellite[9] and Viking spacecraft.[10]

Later on, a generalized distribution function was introduced, having spectral indices r and q, which showed the energetic particles on a broad shoulder and superthermality on the tail of velocity distribution curve of the plasma species termed as (r, q)–velocity distribution function.[11] The (r, q)–distribution is a general form of the kappa distribution and more suitable to describe the plasma particles. In 2015, Abid *et al.*[12] introduced a non-Maxwellian distribution known as Vasyliunas-Cairns (VC) distribution, which has been generated from the kappa distribution[5] and Cairns distribution.[6]

In order to provide a suitable fitting for the magnetosheath electron[13] and solar wind proton[14] data from the AMPTE satellite and CLUSTER, respectively, the non-Maxwellian distribution has been utilized. In both cases, the recorded distribution function diverged from the Maxwellian one, and the broad shoulders, as well as the high energy tails of the observed distributions, corresponded to the protons distribution. The observation documented by the Viking spacecraft[6,15] shows that in the auroral zone, the distribution function of the electron was on the shoulder of the distribution function. In this paper, we have introduced a general form of the non-Maxwellian distribution function generated from Cairns distribution function[6] and (r, q)– distribution function[11] termed as Abid Zubair (*AZ*) – distribution function, which has spectral indices α, r, and q show the rate of energetic particles on the shoulder, energetic particles on a broad shoulder, and superthermality on the tail of velocity

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distribution curve of the plasma species, respectively. The *AZ*– distribution function will be useful to understand that the numerous linear and nonlinear space plasma phenomena occur on the solar wind,[13] magnetosheat,[14] and auroral zone[6,15] more effectively.

II. MAXWELLIAN DISTRIBUTION

The most probable distribution for the plasma particles under the thermal equilibrium is the Maxwell-Boltzmann distribution. Three dimensional Maxwellian distribution can be defined as[2,3]

$$f_J^M(\mathbf{V}_J) = \frac{N_j}{\pi^{3/2}V_{TJ}^3}\exp\left(-\frac{V_J^2}{V_{TJ}^2}\right), \tag{1}$$

where $\mathbf{V}_J$ (N_J) stands for the velocity (density) of the plasma particles while J ($J=e$ stands for electrons, $J=p$ stands for positrons, $J=i$ stands for ions, etc.); $\mathbf{V}_{TJ} = (2T_J/M_J)^{1/2}$ is the thermal speed of the Jth species with $T_J(M_J)$ being the temperature in units of energy (mass).

III. KAPPA DISTRIBUTION

Three dimensional velocity distribution for non-Maxwellian is the kappa distribution function that can be expressed as[5]

$$f_J^\kappa(\mathbf{V}_J) = \frac{N_J C_\kappa}{\pi^{3/2}\kappa_1^3 V_{TJ}^3}\left(1+\frac{V_J^2}{\kappa\kappa_1^2 V_{TJ}^2}\right)^{-(1+\kappa)}, \tag{2}$$

where κ is the spectral index which shows the deviation from the thermal equilibrium to non-thermal equilibrium for space plasmas, while the value of κ must be greater than $3/2$. $C_\kappa = \Gamma(1+\kappa)/\kappa^{3/2}\Gamma(\kappa-1/2)$, Γ represents the gamma function, and $\kappa_1 = \sqrt{(\kappa-3/2)/\kappa}$. It is investigated that the kappa distribution [Eq. (2)] reduces to a Maxwellian distribution [Eq. (1)] for $\kappa \to \infty$.

IV. CAIRNS DISTRIBUTION

The observations were made by the Freja satellite[9] and Viking spacecraft.[10] Cairns *et al.*[6] presented another sort of nonthermal distribution known as Cairns distribution, such as

$$f_J^C(\mathbf{V}_J) = \frac{N_J}{\alpha_1\pi^{3/2}V_{TJ}^3}\left(1+\alpha\frac{V_J^4}{V_{TJ}^4}\right)\exp\left(-\frac{V_J^2}{V_{TJ}^2}\right), \tag{3}$$

where α measures the number of energetic particles within the plasma system under consideration, and $\alpha_1 = 1+3\alpha$. It should be noted that the Cairns distribution [Eq. (3)] reduces to a Maxwellian distribution for spectral index $\alpha = 0$.

V. VASYLIUNAS-CAIRNS (VC) DISTRIBUTION FUNCTION

This generalized distribution (we call it Vasyliunas-Cairns (VC) distribution) for the *jth* plasma species is given as[12]

$$f_J^{VC}(\mathbf{V}_J) = \frac{N_j B_\kappa}{\pi^{3/2}V_{TJ}^3}\left(1+\alpha\frac{V_J^4}{V_{TJ}^4}\right)\left[1+\frac{V_J^2}{\kappa\kappa_1^2V_{TJ}^2}\right]^{-(1+\kappa)}, \tag{4}$$

where B_κ the normalization constant, which depends on the spectral indices (α and κ) and measures the deviation from the thermal equilibrium, is given as

$$B_\kappa = \frac{1}{\kappa_1^3\kappa^{\frac{3}{2}}}\frac{\Gamma(1+\kappa)}{\Gamma(\kappa-1/2)(1+3\alpha)}. \tag{5}$$

We noted that the Vasyliunas-Cairns distribution [Eq. (4)] involves the indices α and κ. It is important to mention that the Vasyliunas-Cairns distribution reduces to: (i) Vasyliunas distribution[5] for $\alpha = 0$, (ii) Cairns distribution[6] for $\kappa \to \infty$, and (iii) Maxwellian distribution for $\alpha = 0$ and $\kappa \to \infty$.

VI. GENERALIZED (r, q) DISTRIBUTION FUNCTION

Consider a generalized (r,q)–distribution function, which is different from the kappa distribution function and the q–distribution function in non-extensive statistics. The (r,q)–distribution function shows a broad shoulder with high energy tails. The deviation from the thermal equilibrium depends significantly on the r and q indices. For the *Jth* species, the (r,q)-distribution function can be expressed as[11]

$$f_J{}^{r,q}(\mathbf{V}_J) = D_{r,q}\left[1+\frac{1}{q-1}\left(\frac{V_J^2}{X_{r,q}V_{TJ}^2}\right)^{1+r}\right]^{-q}, \tag{6}$$

where

$$D_{r,q} = \frac{3E_{r,q}N_J}{4\pi(X_{r,q})^{3/2}V_{TJ}^3}, \tag{7}$$

$$X_{r,q} = \frac{3\Gamma\left(\frac{3}{2r+2}\right)\Gamma\left(q-\frac{3}{2r+2}\right)}{(q-1)^{\frac{1}{r+1}}\Gamma\left(q-\frac{5}{2r+2}\right)\Gamma\left(\frac{5}{2r+2}\right)}, \tag{8}$$

$$\text{and } \mathrm{E_{r,q}} = \frac{(q-1)^{\frac{-3}{2r+2}}\Gamma(q)}{\Gamma\left(q-\frac{3}{2r+2}\right)\Gamma\left(1+\frac{3}{2r+2}\right)}, \tag{9}$$

where $D_{r,q}$ is the constant of normalization that depends on spectral indices r and q. It is confirmed that the (r,q)-distribution function coincides with the kappa distribution function[5] by applying the certain limits such as $r = 0$ and $q = \kappa+1$, and for $r = 0$ and $q \to \infty$, for the Maxwellian distribution[3] recover.

VII. *AZ* DISTRIBUTION FUNCTION

The 3D generalized *AZ*– distribution function is generated from the Cairns distribution function[6] and (r,q)–distribution function,[11] as

$$f_J{}^{AZ}(\mathbf{V}_J) = Y_{AZ}\left(1+\alpha\frac{V_J^4}{V_{TJ}^4}\right)\left[1+\frac{1}{q-1}\left(\frac{V_J^2}{X_{r,q}V_{TJ}^2}\right)^{r+1}\right]^{-q}, \tag{10}$$

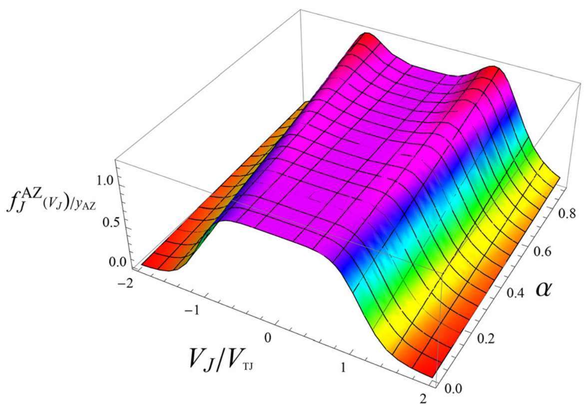

FIG. 1. 3D Plot of the normalized *AZ*-distribution function for the varying spectral index $\alpha(=0.0-0.8)$ with $q=1.6$ and $r=2$.

$$Y_{AZ} = \left(\frac{3N_J}{4\pi V_{TJ}^3}\right)\frac{\rho_{r,q,\alpha}}{X_{r,q}^{\frac{3}{2}}},$$

$$\rho_{r,q,\alpha} = \frac{\Gamma(q)}{(q-1)^{\frac{3}{2r+2}}\Gamma\left(q-\frac{3}{2r+2}\right)(1+3\alpha)\Gamma\left(1+\frac{3}{2r+2}\right)},$$

where Y_{AZ} is the constant of normalization. The *AZ*–distribution function has three spectral indices, α, r, and q, which shows the rate of energetic particles on the shoulder, energetic particles on a broad shoulder, and superthermality on the tail of velocity distribution curve of the plasma species, respectively. It is observed that the spectral indices α, r, and q fulfill the inequalities, i.e., $1 < q$, $\alpha > 0$, and $5/2 < q(r+1)$. The distribution function $f_J^{AZ}(\mathbf{V}_J)$ fulfills the normalization condition, i.e., $\int f_J^{AZ}(\mathbf{V}_J)dV = 1$. It has been examined that (i) the *AZ*-distribution function converts into the (r,q)– distribution [Eq. (6)] for $\alpha=0$, (ii) the *AZ*-distribution reduces to the Vasyliunas Cairns distribution [Eq. (4)] for $r \to 0$, and $q = \kappa + 1$, (iii) the *AZ*-distribution reduces to the Cairns-distribution function [Eq. (3)] for $r \to 0$, and $q \to \infty$, (iv) the *AZ*-distribution reduces to the kappa distribution [Eq. (2)] for $\alpha \to 0$, $r \to 0$, and $q = \kappa + 1$, and (vi) finally, the Maxwellian distribution [Eq. (1)] recover by placing $\alpha \to 0, r \to 0$, and $q \to \infty$ in AZ-distribution.

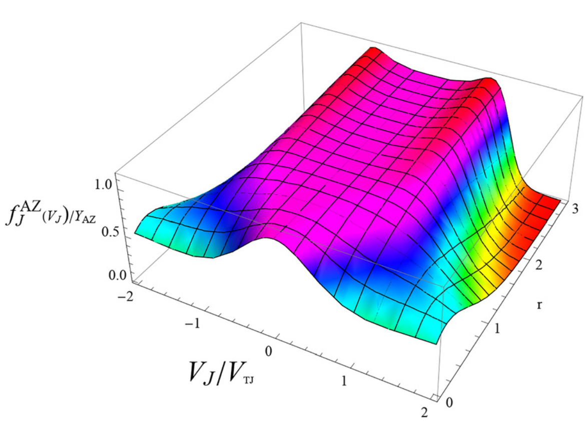

FIG. 2. 3D Plot of the normalized *AZ*-distribution function for the varying spectral index $r(=0.0-3)$ with $\alpha=0.5$ and $q=2.6$.

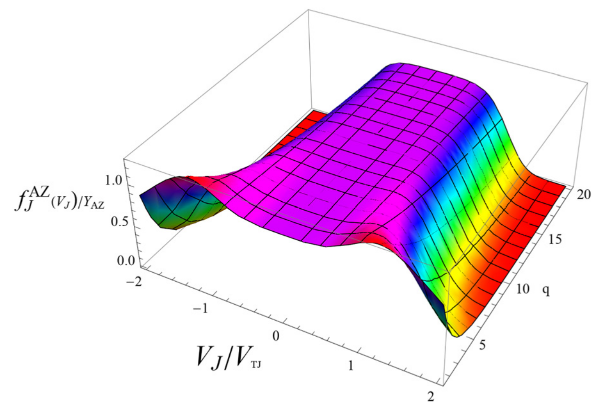

FIG. 3. 3D Plot of the normalized *AZ*-distribution function for the varying spectral index $q(=1.6-20)$ with $\alpha=0.9$ and $r=1$.

VIII. NUMERICAL RESULTS AND DISCUSSION

We numerically studied the generalized *AZ*–distribution function [Eq. (10)] for various effective values of spectral indices α, r, and q. The numerical results displayed in Figures 1–3 show the influence of the three spectral indices α (rate of energetic particles on the shoulder), r (energetic particles on a broad shoulder), and q (superthermality on tail and width) of velocity distribution curve for *AZ*–distributed plasmas. It is observed from Figure 1 that, when we increase the value of the spectral index $\alpha(=0.0–0.8)$ with the fixed value of spectral indices $q(=1.6)$ and $r(=2)$, the high energy particles of the plasma species increase on the shoulder of the velocity distribution curve. Fig. 2 shows the effect of $r(0.0–3)$ for the fixed value of $q(=2.6)$ and $\alpha(=0.5)$. It is studied that the particle distribution on the broad shoulder increases while it decreases on the tail of the velocity distribution curve by increasing the value of spectral index r. Similarly, the effect of $q(=1.6–20)$ with the fixed value of $r(=1)$ and $\alpha(=0.9)$ is shown in Figure 3, where the effect of the high energy particle is more prominent for the low value of $q(=1.6)$, while increasing the q value, the high energy particle on the tail and width of *AZ*–distribution curve decreases accordingly.

In Figures 4–6, we studied a generalized *AZ*–distribution function by applying limits on the three spectral indices (α, r, and q). In Figure 4, the *AZ*–distribution curves indicate that the effect of high energy particles increases on the broad

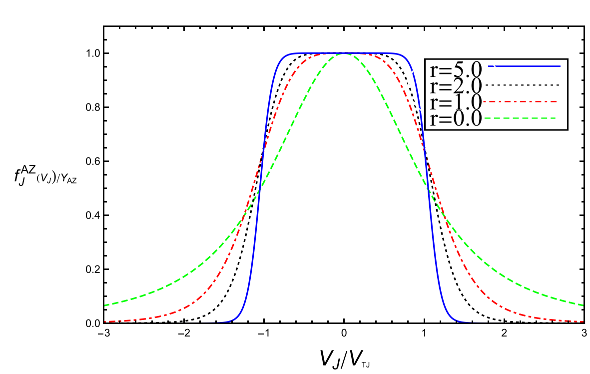

FIG. 4. The normalized *AZ*-distribution function for the varying spectral index $r(=0.0, 1.0, 2.0, 5.0)$ with $\alpha=0.0$ and $q=1.6$.

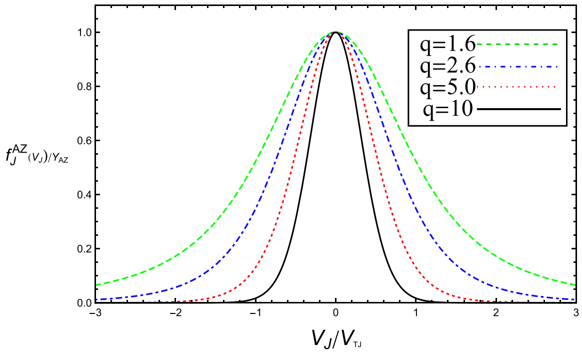

FIG. 5. The normalized *AZ*-distribution function for the varying spectral index $q(=1.6, 2.6, 5.0, 10.0)$ with $\alpha = 0.0$ and $r = 0$.

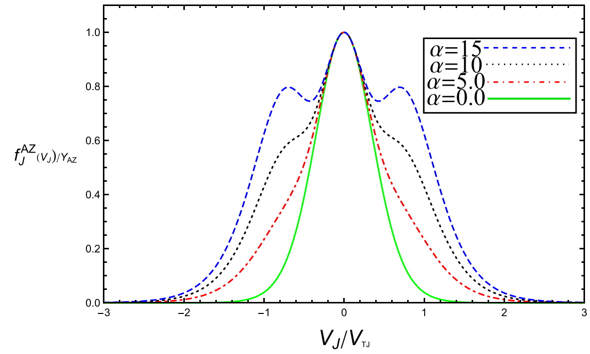

FIG. 6. the normalized *AZ*- distribution function for the varying spectral index $\alpha(=0.0, 5.0, 10.0, 15.0)$ with $r = 0.0$ and $q = 8$.

shoulder while it decreases on the tail of the velocity distribution curves when $r(=0.0, 1.0, 2.0, 5.0)$ increases while keeping $q(=1.6)$ and $\alpha(=0)$ fixed. This is in good agreement with the (r, q)–distribution [Eq. (6)].[11] We noted that the *AZ*–distribution reduces to κ–distribution[5] and the q-distribution functions for $\alpha = 0$ and $r = 0$ are shown in Figure 5. It is observed for the small value of q, high energy tail found similar to kappa distribution $(q = \kappa + 1)$ and q-distribution, but as the value of q (viz., $q = 10$) increases, the result exactly agrees with the Maxwellian distribution.[3] Finally, in Figure 6, the *AZ*–distribution reduces to Cairns distribution[6] for a large value of α, by neglecting the value

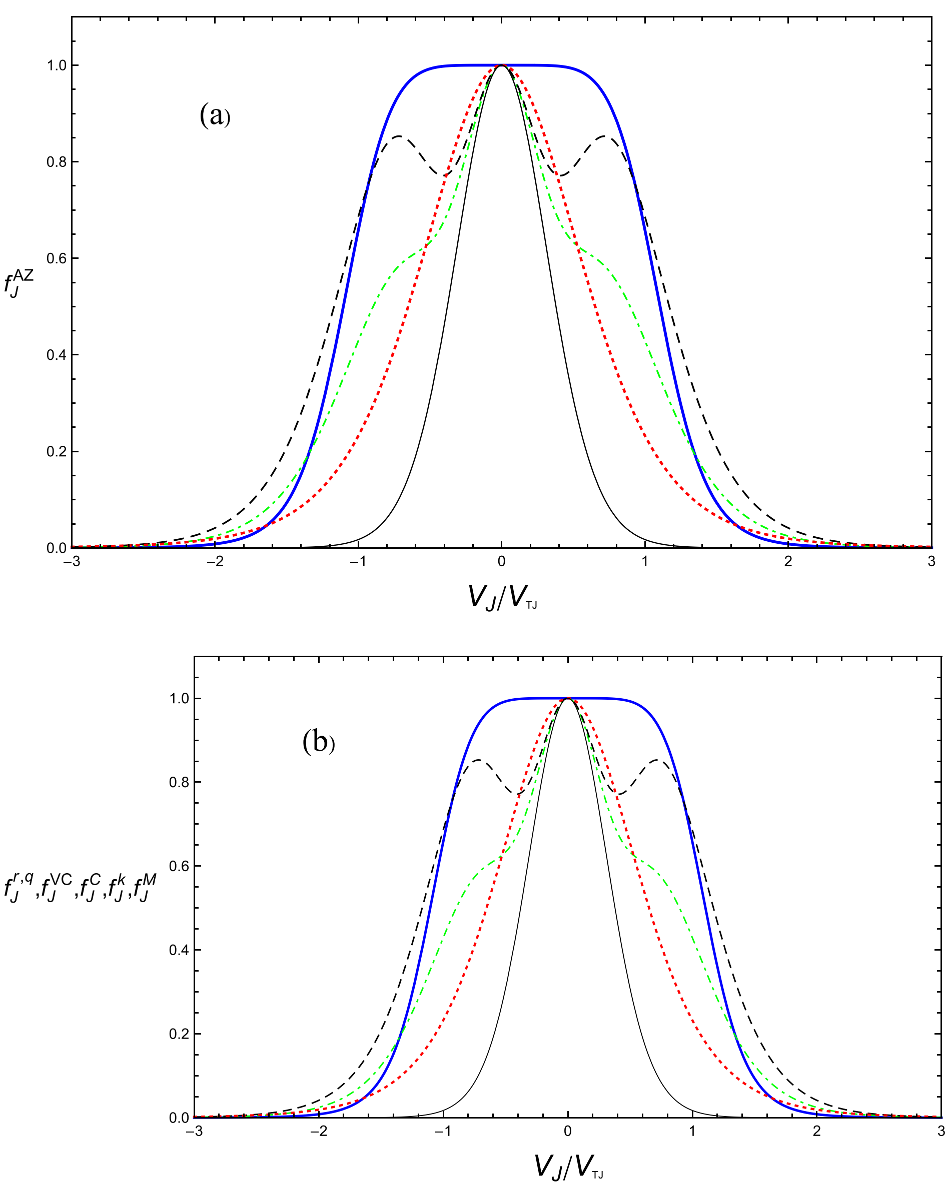

FIG. 7. The normalized *AZ*-distribution function curves, where thick solid blue curve ($\alpha = 0, r = 2$ and $q = 1.6$) corresponds to $(r, q)-$ distribution function, the dashed black curve ($\alpha = 10, r = 0$ and $q = 8$) corresponds to Vasyliunas-Cairns (VC) distribution function, the dotted dashed green curve ($\alpha = 10$, $r = 2$ and $q = 8$) corresponds to Cairns distribution function, the dotted red curve ($\alpha = 0, r = 0$ and $q = 3.6$) corresponds to kappa distribution, and thin solid black curve ($\alpha = 0, r = 0$ and $q = 10$) the curve corresponds to Maxwellian distribution. (b) The normalized (r, q)- distribution function (thick solid blue curve, where $r = 2$ and $q = 1.6$), Vasyliunas-Cairns (VC) distribution curve (dashed black curve, where $\alpha = 0.7$ and $\kappa = 5$), Cairns distribution curve (dotted–dashed green curve, where $\alpha = 0.3$), kappa distribution curve (dotted red curve $\kappa = 4$), and Maxwellian distribution curve (thin solid black curve).

of the spectral index $r(=0)$ and high value of $q(=8)$. It is observed that as the effect of α minimizes, the curve reduces to Maxwellian distribution.[3]

A comparison has been carried out between velocity distribution function curves obtained by AZ-distribution function [Eq. (10)] with those obtained by previous five distribution function [Eqs. (1)–(4) and (6)].The corresponding results are shown in Figure 7. Where the velocity distribution curves obtained from Eq. (10) are shown in Fig. 7(a), where thick solid blue curve ($\alpha = 0$, $r = 2$ and $q = 1.6$) represents the (r, q)–distribution function, the dashed black curve ($\alpha = 10$, $r = 0$ and $q = 8$) represents the Vasyliunas-Cairns (VC) distribution function, the dotted dashed green curve ($\alpha = 10$, $r = 2$ and $q = 8$) represents the Cairns distribution function, the dotted red curve ($\alpha = 0$, $r = 0$ and $q = 3.6$) represents the kappa distribution, and the thin solid black curve ($\alpha = 0$, $r = 0$ and $q = 10$) the curve represents the Maxwellian distribution. Similarly, the velocity distribution function curves plotted from Eqs. (1)–(4), and (6) are shown in Fig. 7(b). It has been investigated that the curves shown in Fig. 7(a), for five different special cases, are similar to those shown in Fig. 7(b).

To summarize, we introduced a more generalized form *AZ*–distribution of non-Maxwellian distribution in space plasmas and defined the basic features of the generalized distribution function. We observed that (i) the *AZ*-distribution function reduces to the (r, q)-distribution for $\alpha \rightarrow 0$, (ii) the *AZ*-distribution function reduces to the *q*-distribution for $\alpha \rightarrow 0$, and $r \rightarrow 0$, (iii) the *AZ*-distribution reduces to Cairns-distribution function for $r \rightarrow 0$, and $q \rightarrow \infty$; (iv) the *AZ*-distribution reduces to Vasyliunas Cairns distribution for $r \rightarrow 0$, and $q = \kappa + 1$; (v) the *AZ*-distribution reduces to kappa distribution for $\alpha \rightarrow 0$, $r \rightarrow 0$, and $q = \kappa + 1$, and (vi) the finally, *AZ*-distribution reduces to Maxwellian distribution for $\alpha \rightarrow 0, r \rightarrow 0$, and $q \rightarrow \infty$.

The investigation of the velocity distributions was observed in the solar wind, planetary magnetosphere and magnetosheaths by the spacecraft, showed that non-Maxwellian distribution of plasma species are very common. There are many space plasma situations, where the velocity distributions of plasma particles show up Maxwellian at low energies, while at high energies show up super thermal power-law tail. The *AZ*-distribution like power law distribution can be used to explain many linear and nonlinear phenomena in space and astrophysical plasma more effectively.[16–36]

There are numerous nonlinear space plasma phenomena[5–8] which can be clarified by non-Maxwellian distribution of plasmas. In this way, one must look for a more generalized distribution function to understand such space plasma system. Therefore, the *AZ*-distribution function will be able to help us to understand such observed space plasma phenomena more accurately.

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