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Velocity distributions of superthermal electrons fitted with a power law function in the magnetosheath: Cluster observations

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[1] Satellite observations have revealed that superthermal electrons in space plasma generally possess a power law distribution. In this paper, we utilize a power law function to model the omnidirectional differential fluxes of superthermal electrons observed by Cluster in the magnetosheath. By assuming an isotropic pitch angle distribution and performing a nonlinear least squares fitting, we can calculate the index α of the power law distribution of the superthermal electrons. We found that in the magnetosheath the indices α of the power law distributions decrease with the increase of ω_{pe}/Ω_e. It is consistent with the results of the recent particle-in-cell simulations, which described the electron distributions scattered by enhanced whistler waves. This is the first reported observation of this relation in space plasma.

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1. Introduction

[2] Space plasma is typically hot, tenuous and collisionless. Its characteristics are essentially controlled by the collective wave-particle interactions rather than those individual particle-particle collisions. Interaction of a small population of superthermal electrons with plasma waves is one of the most fundamental problems of modern plasma physics. Plasma in space generally exhibits a pronounced non-Maxwellian high-energy tail distribution that can be well modeled by a generalized κ or power law function [*Vasyliunas*, 1968; *Feldman et al.*, 1982, 1983; *Summers and Thorne*, 1991; *Collier et al.*, 1996; *Maksimovic et al.*, 1997a, 1997b; *Yin et al.*, 1998; *Saito et al.*, 2000; *Viñas et al.*, 2005; *Xiao et al.*, 2008a; *Schippers et al.*, 2008].

[3] The κ distribution is a full velocity distribution that models both the Gaussian-like, low-energy thermal core particle component, as well as the superthermal tail, where it satisfies a power law function. The formation of κ distributions or power law distributions of superthermal electrons is generally attributed to the interaction of energetic electrons with long-wavelength turbulence [*Gurevich*, 1960; *Tverskoi*, 1968; *Pelletier*, 1982; *Hasegawa et al.*, 1985; *Roberts and Miller*, 1998; *Ma and Summers*, 1998; *Leubner*, 2000]. It was shown that, the κ distributions naturally appear in a procedure of entropy generalization within the frame of nonextensive statistics [*Leubner*, 2004]. This allowed fundamental generalizations of κ distributions including the description of long-range forces in solar wind turbulence [*Leubner and Vörös*, 2005a, 2005b]. *Hasegawa et al.* [1985] obtained an analytical κ distribution solution due to the scattering of a high-intensity radiation field. The κ distributions can also be formed by stationary whistler turbulence [*Ma and Summers*, 1998; *Roberts and Miller*, 1998]. On the basis of Tsallis statistical mechanics, *Livadiotis and McComas* [2009] obtained that the value of κ is larger than 3/2. One thing should be noted is that these approaches are based on a non-self-consistent model. Recently, *Yoon et al.* [2006] developed a self-consistent model, which attributes the electron κ distributions in plasma to the scatter by weak turbulence processes involving the Langmuir/ion sound turbulence and the beam-plasma interaction. *Lu et al.* [2010] investigated the nonlinear evolution of the whistler instability driven by a population of superthermal electrons. They fitted the electron distributions at the quasi-equilibrium stage to a κ function, and found that the spectral index κ of the κ distribution decreases with the increase of ω_{pe}/Ω_e (where ω_{pe} and Ω_e are local electron plasma and cyclotron frequencies, respectively). For energetic electrons, the κ distribution is equivalent to the power law distribution, therefore, we can anticipate that the indices α of the power law distributions will also decreases with the increase of ω_{pe}/Ω_e. The purpose of this paper is to study the relation between the indices α of power law distributions and ω_{pe}/Ω_e by analyzing the electron differential fluxes at high energies, which is measured by Cluster in the magnetosheath.

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2. Fitting Method and Observations

[4] We assume that the superthermal electrons in the magnetosheath satisfy an isotropic power law distribution,

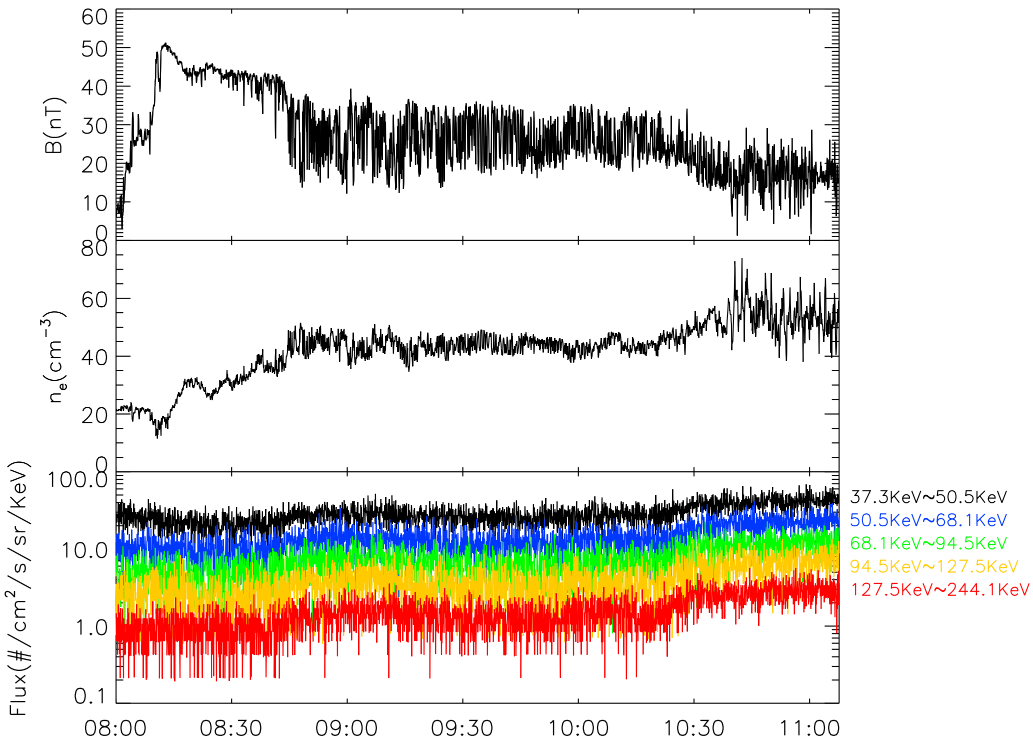

Figure 1. (top) The magnetic field, (middle) the electron density, and (bottom) electron differential fluxes during the interval 0800–1110 UT on 16 March 2002. The spin resolution is 4 s.

which can be described as

$$f(E) \sim E^{-\alpha} \quad (1)$$

where E is the kinetic energy of particles, and α is the index of the power law distribution. The typical values of α in space plasma lie in the range $3.0 < \alpha < 7.0$ [*Vasyliunas*, 1968; *Feldman et al.*, 1982, 1983; *Summers and Thorne*, 1991; *Maksimovic et al.*, 1997b; *Xiao et al.*, 2008b; *Schippers et al.*, 2008].

[5] The relation between the differential flux $j(E)$ and the distribution function is: $j(E) = p^2 f(E)$. In the relativistic limit, we can get

$$p^2 = \frac{m_0 E(E + 2E_0)}{E_0} \quad (2)$$

where $E_0 = m_0 c^2$ is the rest mass energy of particles. Therefore,

$$j(E) = \frac{E(E + 2E_0)}{c^2} f(E) \quad (3)$$

Equation (3), which associates the differential flux $j(E)$ with the electron kinetic energy E, constitutes the basic method to model the observed data. Based on equation (3) and the observed differential fluxes, we can calculate the values of the indices α by performing a nonlinear least squares fitting.

[6] During the interval 0800–1110 UT on 16 March 2002, Cluster was in the high-latitude northern magnetosheath. Cluster consists of four identical spacecraft which maintain a closely separated but evolving spatial array. The same case has also been used by *Gary et al.* [2005] to investigate electron anisotropy constraint in the magnetosheath by assuming that electrons satisfy a bi-Maxwellian distribution. In this paper, we assume that the electron distribution at high energies can be modeled by a power law function, then calculate the index α of the power law distribution by a nonlinear least squares fitting method based on equations (1)–(3). We use measurements from several instruments on Cluster 1. The differential fluxes of energetic electrons are obtained by the Research with Adaptive Particle Imaging Detectors (RAPID) [*Wilken et al.*, 1997]. Magnetic field measurements are from Fluxgate Magnetometer (FGM) experiment [*Balogh et al.*, 2001], and the electron density is obtained from the Plasma Electron and Current Experiment (PEACE) [*Johnstone et al.*, 1997]. These data have 4 s spin resolution.

[7] Figure 1 illustrates the magnetic field, the electron density, and electron differential fluxes from the top to the bottom panels during the interval 0800–1110 UT on 16 March 2002. The spacecraft is moving away from the Earth through the magnetosphere and cusp. In the cusp the magnetic field is weak. At about 0811 UT the spacecraft encounters the magnetopause, and then enters the magnetosheath as indicated by the increase of the magnetic field and decrease of the electron density. In Figure 2, we plot the magnetic field spectra during the interval 0840–1030 UT, when the spacecraft is in the magnetosheath. Obvious wave activity can be found with frequencies around 0.1 Ω_e, which demonstrates the existence of whistler waves. The whistler waves may be excited by electron anisotropic distributions [*Mace*, 1998; *Xiao et al.*, 2006; *Lu et al.*, 2010; *Mace and Sydora*, 2010].

[8] On the basis of equations (1)–(3), we can calculate the values of the indices α of the power law distributions

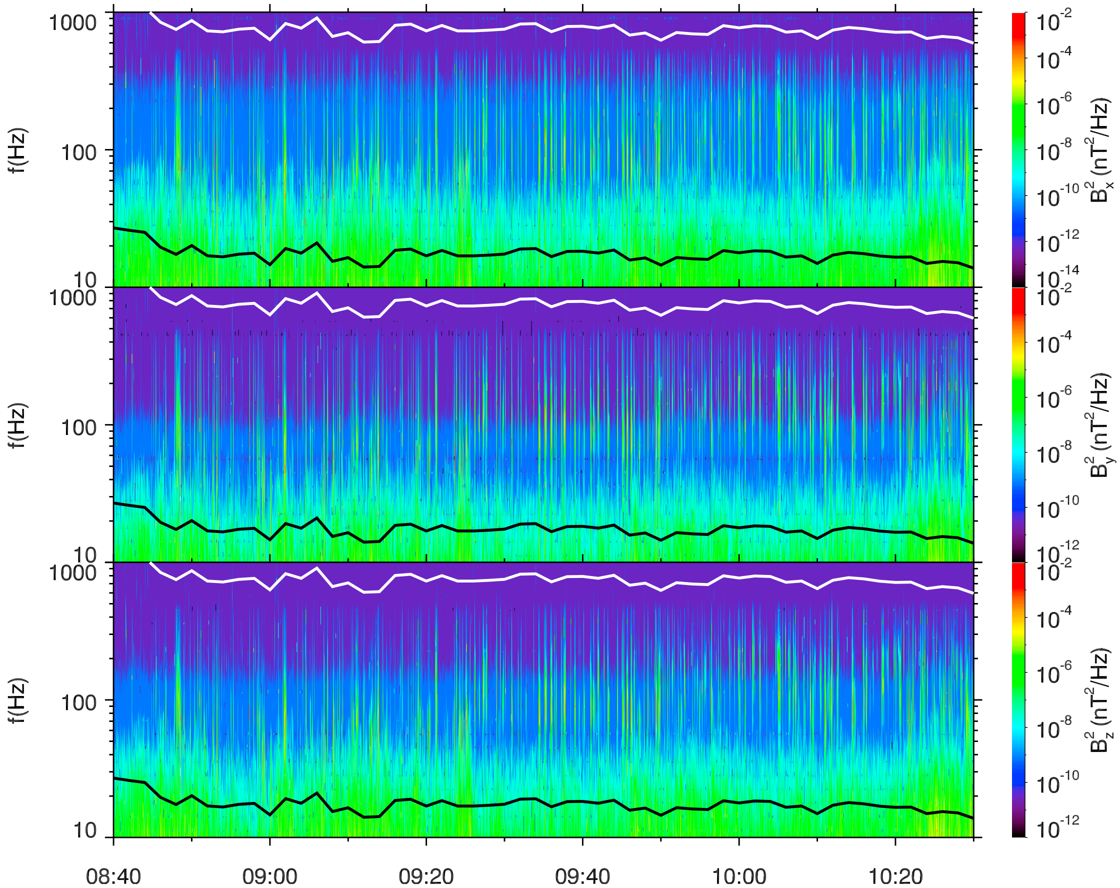

Figure 2. The magnetic field spectra during the interval 0840–1030 UT. The frequency-time spectrogram is measured by the Whisper experiment. The black and white lines represent the local lower hybrid and electron cyclotron frequencies, respectively.

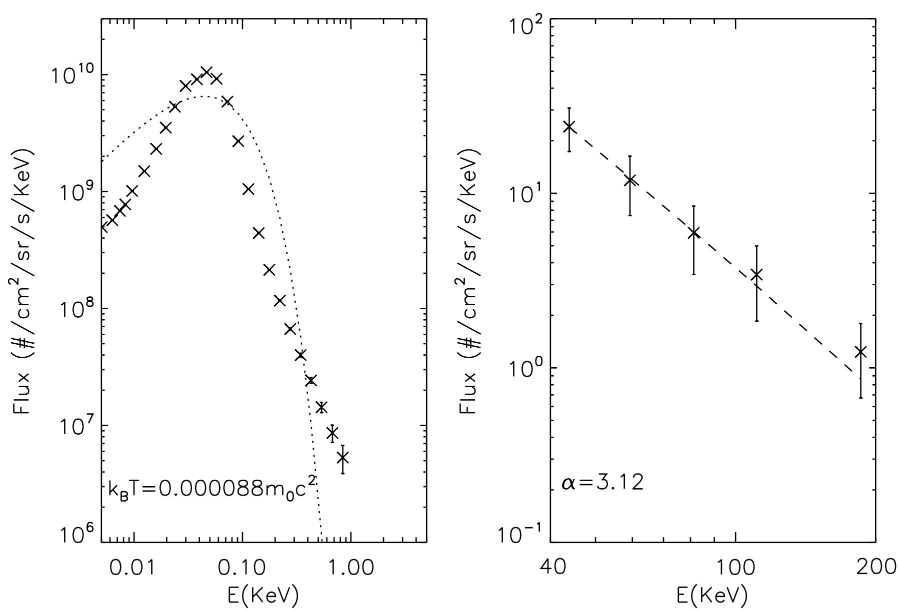

Figure 3. The electron differential fluxes as a function of the electron kinetic energy during the interval 0931–0934 UT for (left) low energy and (right) high energy. The dashed line is fitted by the nonlinear least squares method with LMFIT function in the program Interactive Data Language, which utilizes Levenberg-Marquardt algorithm. The differential fluxes at lower and higher energies are modeled by Maxwellian and power law distributions. The error bars are also plotted.

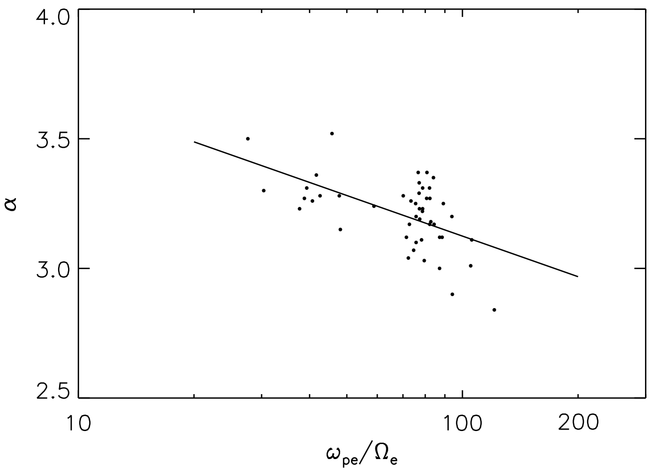

Figure 4. The different values of α (the indices of power law distributions) as a function of ω_{pe}/Ω_e in the magnetosheath. The line can be approximately described as $\kappa \sim -0.226 \ln \frac{\omega_{pe}}{\Omega_e}$.

by a nonlinear least squares fitting method according to the observed electron differential fluxes at high energies by Cluster 1. Figure 3 shows the electron differential fluxes as a function of the electron kinetic energy during the interval 0931–0934 UT for low energies (Figure 3, left) and high energies (Figure 3, right). Both the flux and kinetic energy are the average values during the interval. The dashed line is fitted by the nonlinear least squares method with LMFIT function in the program Interactive Data Language, which utilizes Levenberg-Marquardt algorithm [*Marquardt*, 1963]. In Figure 3 (left), the fluxes with low energies are modeled well by a Maxwellian distribution with $k_B T = 0.000088\ m_0 c^2$. At lower energies below 0.04 keV, the observed fluxes are smaller than the fitted values with a Maxwellian distribution. Therefore, a flat-topped distribution may be better fitted to the observed fluxes. The results are similar to the ISEE 2 observations in the magnetosheath [*Feldman et al.*, 1983]. In Figure 3 (right), the fluxes with higher energies are modeled by a power law distribution. We can find that the value of α is 3.12.

[9] We can further divide the interval 0840–1030 UT into many subintervals, and every subinterval lasts for 2 min. During that interval, there exists obvious whistler wave activity. Then in every subinterval, the index α of the power law distribution can be calculated with the same method as described in Figure 3. Because whistler waves can only scatter energetic electrons, only the power law distribution at high energies is considered. At the same time, the average value of the local ω_{pe}/Ω_e during every subinterval is also different. Therefore, we can obtain the different values of α as a function of ω_{pe}/Ω_e, which is plotted in Figure 4. From Figure 4, we can find that values of α are roughly in the range $2.7 < \alpha < 3.6$, which is consistent with other observations [*Maksimovic et al.*, 1997b]. In addition, we can also find that with the increase of ω_{pe}/Ω_e the values of α decreases. In Figure 4, the line can be approximately described as $\kappa \sim -0.226 \ln \frac{\omega_{pe}}{\Omega_e}$ by the least squares fitting method with LMFIT function in the program Interactive Data Language. The results are consistent with the simulations by *Lu et al.* [2010], which investigated the nonlinear evolution of the whistler instability driven by a population of superthermal electrons. At the quasi-equilibrium stage, the values of α also decrease with the increase of ω_{pe}/Ω_e due to the scattering by the enhanced whistler waves. Please note that the data in Figure 4 show a significant degree of scatter. The reason is that besides ω_{pe}/Ω_e, there are still other factors, such as the amplitude and wave vector of the whistler waves, control the scattering of energetic electrons by whistler waves.

3. Conclusions and Discussion

[10] It is generally considered that superthermal electrons in space plasma possess a power law distribution. In this paper, by assuming that superthermal electrons observed by Cluster in the magnetosheath satisfy an isotropic power law distribution, we can calculate the indices of the power law distributions based on the differential fluxes of superthermal electrons observed by Cluster in the magnetosheath and a nonlinear least squares fitting method. Because the motions of energetic electrons tend to be nonadiabatic, their distributions are usually isotropic [*Wang et al.*, 2010a, 2010b]. The assumption of an isotropic distribution of superthermal electrons is reasonable.

[11] The power law distributions of superthermal electrons have also observed in the magnetosheath by *Feldman et al.* [1982, 1983]. In this paper, we further found that the indices of the power law distributions decreases with the increase of ω_{pe}/Ω_e. The results are consistent with results of particle-in-cell simulations, which investigated the nonlinear evolution of the whistler instability driven by a population of superthermal electrons [*Lu et al.*, 2010]. They also concluded that at the quasi-equilibrium stage the indices of power law distributions of superthermal electrons (or equivalent to values of κ of the κ distributions) decrease with the increase of ω_{pe}/Ω_e due to the scattering by the enhanced of whistler waves. The power law distributions in space plasma are considered as the results of wave scattering. ω_{pe}/Ω_e denotes the importance of scattering by the electrostatic or magnetic part of plasma waves. Therefore, the conclusion that indices of power law distributions of superthermal electrons decreases with the increase of ω_{pe}/Ω_e may have some implications on characteristics of plasma waves in different environments, which need further investigations.

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