On nonresonant proton heating via intrinsic Alfvénic turbulence

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In a recent publication Wu and Yoon [Phys. Rev. Lett. **99**, 075001 (2007)] propose that low-beta protons may be heated by turbulent Alfvén waves via nonresonant wave-particle scattering. The present Brief Communication clarifies some conceptual issues and describes the theoretical methods adopted in the above reference in more detail. © *2009 American Institute of Physics*.
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In a recent publication Wu and Yoon[1] propose that low-beta protons may be heated by turbulent Alfvén waves via nonresonant wave-particle scattering. This is an important issue in view of the as-yet-unexplained origins of hot solar and stellar coronae.[2,3] The discussion in Ref. 1 is based on quasilinear theory[4,5] and the process is mainly applicable for low-beta protons. The present brief communication is a result of private communications with our colleagues, who led us to realize that the terse discussion in Ref. 1 did not completely succeed in spelling out the most salient points. We also realize that we should have clarified some conceptual issues and have described the theoretical method in more detail.

The purpose of this Brief Communication is to clarify what it means by "heating," to further elaborate on the new concept introduced in Ref. 1 and to further explain the theory and method adopted in Ref. 1. Let us begin with the meaning of heating. By definition, plasma heating represents an enhancement in "average kinetic energy" or "temperature." However, temperature is meaningful only if the microscopic motion of the particles is stochastic. In this sense, an increase in the average kinetic energy is not necessarily equal to heating. Taking the moment of the particle velocity distribution function can be misleading since in some cases the increase in the kinetic energy may be owing to bulk fluid motion rather than randomized individual particle motion. Another important aspect is that heating must be physically irreversible. It is customary to regard the increase in thermal energy as a result of the dissipation process where a certain type of energy is partially converted to heat. In short, it is customary to regard true heating as satisfying two criteria, namely, stochastic motion of the particles and some sort of dissipation (meaning irreversibility).

The heating process discussed in Ref. 1, however, does not involve dissipation. While the proton motion regarded in Ref. 1 is "random," it is so only by virtue of the fact that the particle motion is "parasitic" to the turbulent nature associated with the waves—see the analytic expression (10) in Ref. 1. Subsequent numerical work carried out in Ref. 6 demonstrates that the so-called heating process discussed in Ref. 1 is reversible in that when the turbulence subsides, the temperature diminishes and returns to the initial value. In view of these, Ref. 6 coined the term "pseudoheating" to describe the enhanced average kinetic energy. Alternatively, the increase in average particle kinetic energy discussed in Refs. 1 and 6 can be viewed in terms of an "apparent temperature."

To recapitulate, the theory discussed in Ref. 1 suggests that when turbulent Alfvén waves attain a high energy density in natural plasmas, these waves can result in a high apparent temperature. However, such a process must not be confused with the customary thermodynamic heating that involves irreversibility and dissipation.

The second point is that while the notion of kinetic energy density of particle motion induced by waves is not new, Ref. 1 made a specific application of this concept for low-beta protons reacting to turbulent Alfvén waves. In general, the particle kinetic energy density induced by an arbitrary wave mode in magnetized plasmas is given by the formal expression[7,8]

$$\int d\mathbf{k} W_P(\mathbf{k}) = \int d\mathbf{k} a_i(\mathbf{k}) \left\{ \frac{\partial}{\partial \omega_{\mathbf{k}}} [\omega_{\mathbf{k}} \epsilon_{ij}(\omega_{\mathbf{k}}, \mathbf{k})] - 1 \right\} \times \frac{|E_{\mathbf{k}}|^2}{8\pi} a_j^*(\mathbf{k}), \quad (1)$$

where $\epsilon_{ij}(\omega_{\mathbf{k}}, \mathbf{k})$ is the linear dielectric response tensor and $a_i(\mathbf{k})$ denotes the polarization vector. Equation (1) reflects the fact that the computation of the induced particle kinetic energy density in general cannot be derived on the basis of intuitive discussions. For low-frequency Alfvén waves, however, a substantial simplification of Eq. (1) can be made, and the result is given by

$$\int d\mathbf{k} W_P(\mathbf{k}) = \int d\mathbf{k} \frac{c^2}{v_A^2} \frac{|E_{\mathbf{k}}|^2}{8\pi} = \int d\mathbf{k} \frac{|B_{\mathbf{k}}|^2}{8\pi}. \quad (2)$$

Here $v_A = B/\sqrt{4\pi n_p m_p}$ stands for Alfvén speed, n_p and m_p being the proton number density and proton mass.

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Indeed, for this particular case one could alternatively derive the above result based on a simple fluid theory. From the available fluid theory one could easily derive the velocity perturbation $\delta\mathbf{v}$ responding to magnetohydrodynamics (MHD) Alfvén wave magnetic field perturbation $\delta\mathbf{B}$ (Ref. 9),

$$\delta\mathbf{v} = \frac{\delta\mathbf{B}}{\sqrt{4\pi n_p m_p}}. \tag{3}$$

Correspondingly the kinetic energy density is

$$\frac{1}{2}n_p m_p \delta v^2 = \frac{\delta B^2}{8\pi}. \tag{4}$$

In case there exist other discrete coherent Alfvén waves simultaneously in the system that may also induce fluid motion, then intuitively the spectral representation of Eq. (3) may be treated by the superposition of Fourier spectral components,

$$\delta\mathbf{v} = \sum_{\mathbf{k}} \frac{\mathbf{B}_{\mathbf{k}}}{\sqrt{4\pi n_p m_p}}. \tag{5}$$

In this case we have

$$\frac{1}{2}n_p m_p \delta v^2 = \sum_{\mathbf{k}} \frac{B_{\mathbf{k}}^2}{8\pi}, \tag{6}$$

where a spatial average has been taken.

Note that both Eqs. (6) and (10) in Ref. 1 are seemingly identical and compatible with Eq. (2). However, a crucial distinction should be made between these two results. The kinetic energy density described by Eq. (6) is associated with the bulk fluid motion, and therefore it does not represent a thermal energy density. Consequently, it has nothing to do with heating. As noted already, true heating must involve randomization of individual particle motion. In fluid theory the notion of individual particle motion is absent. On the other hand, in Ref. 1 we show that there is an apparent temperature associated with random proton motion which possesses a Maxwellian distribution without bulk motion.

Finally we discuss the third point. In Ref. 1 several essential assumptions are made at the outset: among them is that the wave frequency ω_k and proton parallel velocity $v_\parallel$ satisfy the following inequality:

$$\Omega_p \gg \omega_k \gg k v_\parallel, \tag{7}$$

where $\Omega_p = eB/m_p c$ is the proton gyrofrequency. Most importantly the turbulent Alfvén waves are treated as intrinsic and their spectral energy density is in general slowly varying with time. This last point deserves further elaboration, which is given below. Conceptually Alfvén waves under consideration are implicitly excited by a certain "source mechanism." For instance, a certain kinetic instability attributed to a small population of "energetic ions" may be operative. In such a situation, thermal protons may determine the wave dispersion relation, whereas the growth (or damping) rate is mainly dictated by the source ions. Since the dynamics of each species may be described separately, Ref. 1 neglects the discussion of instability but simply assumes that the growth rate exists so that time dependence of the wave energy is known—see the Appendix for further details.

In closing we reiterate that the primary objective of this brief communication is to clarify issues that are related to Ref. 1. Hence in the present discussion we shall not review or cite other publications on quasilinear theory which may be relevant and interesting from a general viewpoint.

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APPENDIX: THEORETICAL ISSUES

Quasilinear kinetic theory is widely used in the study of solar wind. For a recent review of quasilinear kinetic theory in the context of solar physics, see Ref. 10. However, most of the existing theories rely on resonant wave-particle processes. Reference 1, on the other hand, emphasized that nonresonant interaction of Alfvén waves and protons may lead to the apparent heating of low-beta protons. In the present Appendix, we supplement the discussions presented in Ref. 1.

Let us consider three species of particles: thermal protons, electrons, and a tenuous population of "source" ions. The tenuous population of energetic ions is responsible for the excitation of Alfvén waves via cyclotron resonance. We denote the distribution functions of these particles by F_j, where $j = p, e, s$ stand for protons, electrons, and source ions, respectively. Although electrons do not play any role in the quasilinear theory, it is necessary to include them in the dispersion equation, which takes the form

$$\omega^2 = c^2k^2 - \omega\sum_j \frac{\omega_{pj}^2}{2}\int d\mathbf{v}\frac{v_\perp}{\omega \pm \Omega_j - kv_\parallel} \times \left[\frac{kv_\perp}{\omega}\frac{\partial F_j}{\partial v_\parallel} + \left(1 - \frac{kv_\parallel}{\omega}\right)\frac{\partial F_j}{\partial v_\perp}\right], \tag{A1}$$

where ω_{pj} and Ω_j are the plasma frequency and gyrofrequency defined for species j, respectively. If we define the solution of Eq. (A1) by $\omega(k) = \omega_k + i\gamma_k$, where ω_k is the real frequency and γ_k is the growth rate, then the real frequency ω_k can be determined by neglecting the low-density source ions. These ions are important only for the discussion of the growth rate γ_k. Thus we have

$$\omega_k^2 = c^2k^2 - \omega_k \sum_{j=e,p} \frac{\omega_{pj}^2}{2} \int d\mathbf{v} \frac{v_\perp}{\omega_k \pm \Omega_j - kv_\parallel} \frac{\partial F_j}{\partial v_\perp}. \quad \text{(A2)}$$

Equation (A2) leads to the desired dispersion relation $\omega_k = kv_A$ upon considering

$$|\Omega_e| \gg \Omega_p \gg \omega_k \geq kv_\parallel. \quad \text{(A3)}$$

On the other hand, the growth rate γ_k may be approximately calculated by a separate equation

$$\gamma_k = \frac{\pi}{2} \frac{\omega_{ps}^2}{2} \int d\mathbf{v} v_\perp \delta(\omega_k \pm \Omega_s - kv_\parallel) \times \left[\frac{kv_\perp}{\omega_k} \frac{\partial F_s}{\partial v_\parallel} + \left(1 - \frac{kv_\parallel}{\omega_k}\right) \frac{\partial F_s}{\partial v_\perp} \right], \quad \text{(A4)}$$

where the contributions from the background electrons and protons are ignored. These particles cannot resonate with Alfvén waves, hence they have no contribution. Alfvén waves can be excited if the source ions possess a distribution that satisfies

$$\left[\frac{kv_\perp}{\omega_k} \frac{\partial F_s}{\partial v_\parallel} + \left(1 - \frac{kv_\parallel}{\omega_k}\right) \frac{\partial F_s}{\partial v_\perp} \right]_{v_\parallel = |v_A \pm \Omega_s / k|} > 0. \quad \text{(A5)}$$

A prime example is when the source ions form a beam with average speed much higher than the Alfvén speed. In Ref. 1 we postulated that the discussion of the wave excitation is beyond the scope of the paper. That is, we assumed that γ_k is known. Here, we have supplemented our earlier discussion.

According to quasilinear theory we may write the two separate kinetic equations for F_p and F_s as follows:

$$\frac{\partial F_p}{\partial t} = \frac{1}{8\pi n_p m_p v_A^2} \int d\mathbf{k} \frac{\partial B_k^2}{\partial t} \frac{1}{v_\perp} R(v_\perp R F_p),$$

$$\frac{\partial F_s}{\partial t} = \frac{\pi e^2}{2m_s^2 c^2} \int d\mathbf{k} B_k^2 \frac{1}{v_\perp} R[\delta(\omega_k \pm \Omega_s - kv_\parallel) v_\perp R F_s], \quad \text{(A6)}$$

$$R = (v_A - v_\parallel) \frac{\partial}{\partial v_\perp} + v_\perp \frac{\partial}{\partial v_\parallel}.$$

In deriving the first equation in Eq. (A6) we have made use of the relation $2\gamma_k B_k^2 = \partial B_k^2 / \partial t$. The discussion of the second kinetic equation in Eq. (A6), namely, the quasilinear diffusion equation for the source ions, and the growth rate (A4) is omitted in Ref. 1 because it is unimportant to the essence of the theory.

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