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A theory of heating of quiet solar corona

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A theory is proposed to discuss the creation of hot solar corona. We pay special attention to the transition region and the low corona, and consider that the sun is quiet. The proposed scenario suggests that the protons are heated by intrinsic Alfvénic turbulence, while the ambient electrons are heated by the hot protons via collisions. The theory contains two prime components: the generation of the Alfvénic fluctuations by the heavy minor ions in the transition region and second, the explanation of the temperature profile in the low solar atmosphere. The proposed heating process operates continuously in time and globally in space. © 2015 AIP Publishing LLC.

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I. INTRODUCTION

It is well known that the sun has a hot corona but it is not such a common knowledge that the sun has a relatively cool chromosphere that separates the corona and the surface of the sun.¹ The surprising fact, as depicted in Fig. 1, is that the temperature profile varies mildly until it gets close to the chromosphere-corona transition region, which has a thickness of only several hundred kilometers. As shown in Fig. 1, the temperature jumps from below several hundred thousand degrees to a couple of million degrees (Kelvin) within a narrow spatial region. The schematic plot (Fig. 1) is largely based on the result acquired with the Skylab observation.^{2,3} It leaves an impression that the transition region is shielding the chromosphere from the overlying hot corona. This phenomenon has baffled many scientists and attracted immense theoretical interest since its discovery several decades ago, and yet, the physical origin of the observed heating process is still unexplained. This is despite the fact that a number of hypothetical models with numerical results are discussed in the literature. Here we list several publications for examples.⁴⁻⁷

The purpose of the present paper is to propose a theory, which is dedicated to the low corona. The scientific objective is to understand the underlying physics responsible for the heating rather than to stress quantitative comparison with observations. In our theory, we use kinetic theoretical approach instead of fluid dynamics because of the following considerations.

Conceptually, fluid theory studies mainly “transport and conduction of thermal energy.” It does not serve the purpose if we want to understand the “physics of heating.” If we want to study effects on thermal energy due to dissipation, *ad hoc* hypothetical model is inevitable because collisions

are absent. Consequently, the physics becomes obscure. In plasma physics, heating is a topic that is far from understood. As a result, it is rarely discussed in standard textbooks. The fundamental difficulty comes from the fact that in general, inter-particle collisions are infrequent in plasmas. For example, in a neutral gas, collisions can thermalize particles when it is compressed. However, the same process does not occur in a (largely collisionless) plasma.

There is another fundamental theoretical difficulty. In fluid dynamic theory, an essential assumption is always used implicitly: the medium should be near thermal equilibrium. This condition is necessary because only when it is true that the temperature and some other physical quantities are meaningfully defined. For a neutral gas, this condition is in general satisfied because collisions can quickly thermalize particles. However, plasmas are different because they generally have very low collision frequency so that they are

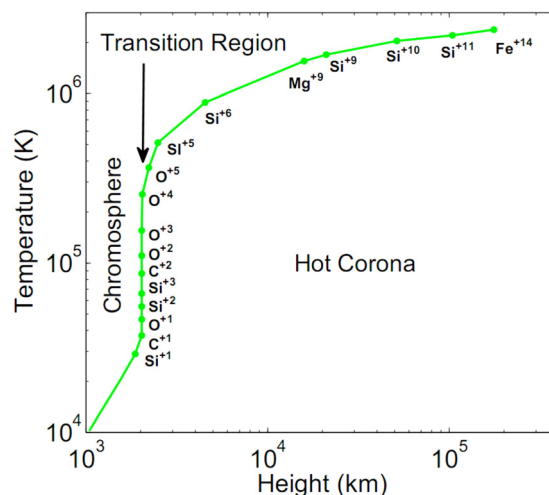


FIG. 1. A schematic description of the solar atmosphere at low altitudes. It is largely based on the observation made with the Skylab mission. The figure gives us an overall picture of how the temperature varies in the transition region and the low corona.

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often in non-equilibrium states. The pickup ions in the solar wind^{8,9} are a good example for which fluid dynamical description would yield meaningless results. From this viewpoint, the physical meaning of temperature discussed in plasma fluid theory is often unclear. In this sense, plasma heating is a difficult and tricky issue.

Now let us turn our attention to another issue. Recent observations of Alfvén waves in the solar atmosphere with Hinode satellite have greatly interested scientists.^{10–14} It is found that MHD Alfvén waves seem to carry a large amount of energy. However, most of these waves have wave periods 100s (and longer) and wavelengths (several hundred kilometers and longer). It is very difficult, if not impossible, to show theoretically or intuitively that heating by such MHD waves can take place. Moreover it is not compatible with the steep temperature gradient inside the transition region.

In solar physics and astrophysics literature, Alfvén waves are usually discussed in the context of MHD theory.¹⁵ It is often conjectured that dissipation of Alfvén waves might result in heating. On the other hand, in plasma physics, it is also known that dissipation of wave energy does not necessarily mean that the absorbed wave energy would be converted to thermal energy of the particles, unless it can be specifically demonstrated. Thus, it is difficult to conceive that MHD Alfvén wave can result in heating.

Figure 1 is interesting for two reasons. It clearly manifests that the coronal heating process begins in the transition region. At the same time, it also shows that most of the heavy minor ions are born in this region. The conventional explanation is that the various species of minor ions are produced due to collisional ionization of neutral atoms with hot electrons having appropriate and sufficient energies. In this paper, we suggest that there is an important implication: the heavy ions may play a crucial role in the heating process.

In the more recent literature, there are discussions in which emphasis is placed on heating via ion cyclotron resonance. Physically, this is a more attractive notion than the traditional thought, which conjectures heating due to dissipation. The basic idea is described as follows: Owing to the fact that the ambient magnetic field decreases progressively in the corona, high-frequency, Alfvén waves evolve gradually into ion cyclotron waves. As a result, ion cyclotron resonance may become an important process for heating. The process is extensively discussed in a number of publications and we just list a few for example.^{16–19} However, wave-particle resonance is a highly selective process. For a given ion gyro frequency, only a fraction of the ambient ions can participate in the process. Of course, spatially the magnetic field varies gradually so that more ions can be affected eventually if the plasma has already attained a high flow speed.

Following several years worth of theoretical research, we have explored several new areas in plasma physics: (a) effects of turbulent Alfvén waves on proton temperature,²⁰ (b) heating of ions via non-resonant interactions with Alfvénic fluctuations,^{21,22} (c) spontaneous generation of Alfvénic fluctuations by solar minor ions,²³ and (d) induced heating by Alfvénic fluctuations.²⁴ On the basis of this newly acquired knowledge, we now propose a scenario in which the essential physical perception is fundamentally different

from what has been discussed in the existing literature. The purpose of this paper is to report the proposed scenario. In the present approach, kinetic theory is employed. We shall place emphasis on the discussion of basic physics rather than quantitative comparison with observations. We share the same view with many theorists who have done pioneering research that the Alfvén waves are important. However, we are interested in broad-band turbulent fluctuations rather than coherent MHD Alfvén waves.

We reiterate that in this article, we shall restrict our discussion to the region that is depicted in Fig. 1. In this low-altitude region, the plasma has little bulk motion, as known from observation²⁵ as well as on the basis of numerical modeling,²⁶ so that the heating process is well defined without being affected by the kinetic energy associated with the bulk motion of the plasma.

In the present discussion, we propose a scenario in which the heavy minor ions in the low altitude solar atmosphere may be an important element. The rationales are (a) these minor ions are created in the region where the observed heating process starts, (b) the newborn minor ions, which are eventually either annihilated or removed but they are also replenished, are self-sustaining, and (c) the heavy minor ions can spontaneously generate enhanced Alfvénic fluctuations, and (d) together with other preexisting narrow band Alfvén waves they can effectively thermalize ambient protons, as to be clarified later.

The organization of this presentation is as follows: Section II explains and clarifies some basic concepts, which are essential and critical in our theory. Section III presents an overview of the proposed scenario before we move on to Sec. IV in which the discussion involves some ramifications of the theory. In Sec. V, the discussion focuses on the main issue: explanation of the temperature profile.

II. GENERAL CONSIDERATIONS

A. Heavy minor ions

The present theory discusses the corona of a quiet sun because the corona is best defined in such a case. Observations find that it is nearly spherically symmetric and stationary. It implies that the heating mechanism operates not only globally but also self-sustainingly. Moreover, since observations also show that the heating process actually starts in the transition region, as shown in Fig. 1, we argue that this is an important clue. As a result, we are attracted to the heavy minor ions and suspect that they may be important in the heating process. We note that not only do they exist everywhere in the low solar atmosphere but also they are created in the same region where heating takes place.

The existence of the minor species in the solar atmosphere was known in solar physics since very early on.¹ However, the implications were not clear in the early years. The discovery of the emission lines in the spectra of Fe XIV and Fe X and other findings led scientists to conceive that a hot corona may exist, but the connection that the heavy ions could play a significant role in the heating process has not been made hitherto. This belief is largely due to the fact that the abundance of each species is very low.¹ However, we may have underestimated the importance of the minor ions.

First, the number of ion species actually in existence is very large. [Note that the number of heavy ions displayed in Fig. 1 only represents a tiny fraction of the actual ion species.] Their cumulative and resultant effects may be actually significant.²⁷ In the theory to be discussed, the broadband Alfvénic fluctuations generated by the heavy ions are critical and important. The gyro-frequencies of the large number of species of the heavy ions can form a broad spectrum which justifies the validity of the quasilinear theory approach used in our discussion.

B. Spontaneous generation of Alfvénic fluctuations

Spontaneous generation of high-frequency electromagnetic waves is well known in radiation theory.²⁸ In general, spontaneous emission takes place for many other wave modes in the plasma, but in the customary literature only Langmuir waves in unmagnetized plasmas^{29,30} are studied. Recently, we investigated the case of Alfvénic fluctuations.²³ The study is solely for the solar atmosphere where a huge number of heavy ions exist. These ions have different ion gyro-frequencies that can cumulatively form a broad frequency spectrum. This is one of the requirements in the theory of turbulent heating, which we are proposing. The underlying physics of the spontaneous generation process is that when those ions, which can resonate with Alfvén waves, are interacting, they may transfer a small fraction of their kinetic energy to the Alfvén waves. A theory²⁴ is developed on the basis of this notion. A brief review of the essence of the theory is given in Sec. III.

C. Some basic and necessary concepts

Plasma waves of a given type of mode may exist in very different forms. In some case, they appear as narrow wave packets but in other case, they may emerge as a spectrum of turbulent waves for which wave-particle interactions are important. The former may be treated as coherent waves so that they are usually considered in fluid theory while the latter should be treated with kinetic theory. If we are only interested in the linear stability, the usual Vlasov theory is sufficient. However, strictly speaking, turbulent waves have random phases and thereby the ensemble average of the wave fields vanishes. To avoid possible confusion, in this paper, we define “intrinsic turbulent Alfvén waves” as the sum of Alfvénic fluctuations due to heavy ions, in contrast to other kinds of intrinsic Alfvén waves, which are due to local disturbance or energetic particles. The fluctuations are generated by the heavy ions via spontaneous process. Since this process is newly discussed, it is elaborated below.

In kinetic theory, the wave properties are discussed on the basis of the kinetic equation for the spectral energy density W_k , defined by

$$W_k = \frac{\langle |\delta B|^2 \rangle_k}{8\pi}, \quad (1)$$

where δB denotes the fluctuating magnetic field and $\langle \rangle_k$ is the spectral representation of the correlation function. In the solar atmosphere, there are many species of heavy ions,

which as will be shown, can generate Alfvénic fluctuations. In the discussion, we consider that they are cold and each ion species can only resonate with Alfvén waves in a narrow frequency range. If the corresponding spectral energy is denoted as $W_k^{(s)}$, then the resultant spectral energy may be expressed as the sum of spectral energies generated by all ion species. We also assume that the spectral energies of coherent Alfvén waves have frequencies below the spectrum of the ion gyro-frequencies Ω_s so that they are non-resonant with the heavy ions. Thus they may be lumped into one component and denoted by W_k^0 , which is considered to be arbitrary. Hence, in our theory, we express the total spectral energy of the Alfvén waves as

$$W_k = W_k^0 + \sum_{s \neq p, h} W_k^{(s)}. \quad (2)$$

[In Eq. (2), the exclusion of protons (p) and helium ions (h) in the sum is due to the fact they cannot participate in the process of spontaneous generation because they cannot resonate with the Alfvén wave.] Although the existence of W_k^0 is practically expected in real solar atmosphere, in our theory, however, it is indeterminate. Physically, $\sum_s W_k^{(s)}$ is important mainly because of the spectral width, which is crucial in the quasilinear theory to be used for discussing the heating process of interest. We remark that since the heating process only depends upon non-resonant wave-particle interactions, the functional form of the spectral energy is irrelevant. As a matter of fact in the following discussion, only the total wave energy density, which is defined below, is important and relevant

$$\begin{aligned} \frac{\delta B_w^2}{8\pi} &\equiv \int d^3k W_k = \int d^3k W_k^0 + \sum_{s \neq p, h} \int d^3k W_k^{(s)} \\ &\equiv \frac{\delta B_{0w}^2}{8\pi} + \sum_{s \neq p, h} \frac{\delta B_{sw}^2}{8\pi}. \end{aligned} \quad (3)$$

Equation (3) is introduced only for conceptual discussion. The contribution of each species of the heavy ion to the total wave energy density may be small but the sum of hundreds of species can be significant because the spectrum of $W_k^{(s)}$ is broad in \mathbf{k} space. In our discussion, δB_w^2 is important throughout the discussion. Eventually, the ratio $\delta B_w^2/B_0^2$ (where B_0 denotes the ambient magnetic field) will appear as an essential parameter in our theory.

D. Thermalization of ions by Alfvénic fluctuations

In general, the thermodynamic state of a system may be affected by intrinsic physical conditions. Since in the solar atmosphere Alfvénic fluctuations preexist, an issue arises: can the intrinsic turbulence affect the thermodynamic quantities. On the basis of quasilinear theory, we find that in low-beta plasmas, Alfvénic turbulence may result in heating.^{21,22} The process may be simply outlined as follows: If we consider that in the absence of the Alfvén waves the ambient protons have a Maxwellian velocity distribution with a temperature T_0 , then the proton temperature is modified by the Alfvénic turbulence such that perpendicular temperature $T_{p\perp}$ becomes

$$T_{p\perp} = T_0 + \frac{\delta B_w^2}{8\pi n_p}, \quad (4)$$

where n_p denotes the proton density and $\delta B_w^2/8\pi$ which denotes the energy density of the Alfvénic turbulence which is defined early by Eq. (3). The implication of Eq. (4) is that not only can the intrinsic Alfvén turbulent waves induce random motion of the ions but actually the waves can thermalize them so that the ions can attain a higher temperature. Apparently, Eq. (4) implicates a heating process. Since the theory presented in Refs. 21 and 22 does not involve dissipation, some controversies ensued. It turns out, however, that the concern is unwarranted, as elucidated in Ref. 24.

III. AN OVERVIEW OF THE PROPOSED SCENARIO

It is useful to present an overview of the proposed scenario before the details of the theory are described in Sec. IV. The basic notion is that we consider a quasi-stationary system at the outset. In such a system, intrinsic Alfvénic turbulence preexists. By Alfvénic turbulence in the present discussion, we mean that it has two primary constituents: first, the broadband Alfvénic fluctuations generated pervasively by the heavy ions and second, coherent Alfvén waves which are mostly excited by local disturbance, energetic particles, or instability. The fluctuations are important in view of the fact that the waves have random phases while the coherent waves are also important because they can significantly increase the total wave energy density.

The essential postulate imposed in our theory is that the total wave energy density defined in Eq. (3) is conserved in the phenomenon. In the proposed scenario, there are four essential steps: (a) immediately after their creation, the heavy ions are affected by the intrinsic Alfvénic turbulence by non-resonant interaction, (b) the heavy ions can then generate Alfvénic fluctuations spontaneously so that the intrinsic waves are replenished, (c) the turbulence heats the ambient protons, and (d) the hot protons in turn heat the ambient electrons by collisions and consequently the heated electrons result in collisional ionization of neutral atoms. Then the process undergoes repetitions, thus establishing stationary process.

Finally, let us make one remark. The typical thickness of the transition region is about several hundred kilometers. In the theory we consider that the proton gyro frequencies are in the range ($1 \sim 10^2$) kHz and the plasma density is about 10^8 cm^{-3} . For these parameters, the intrinsic Alfvén waves of interest have wavelengths shorter than a kilometer. Hence the local approximation is justified. It is based on this consideration that the kinetic equation describing the wave spectral energy is derived. As a result, the local theory is applied to discuss the profile of the temperature across the transition region as well as the low corona.

IV. RELEVANT BASIC THEORIES

A. Generation and saturation of Alfvénic fluctuations

In a preceding article,²³ we discuss the generation of Alfvénic fluctuations by minor ions in the solar atmosphere. We show that heavy minor ions can generate Alfvénic

fluctuations which consist of a spectrum of plane waves, while each wave satisfies the Alfvén wave dispersion relation, $\omega_k = k_z v_A$, where v_A is the Alfvén speed and k_z is wave vector component parallel to the ambient magnetic field. To avoid any confusion, we reiterate two basic points that are assumed in the discussion. First, the heavy minor ions are newly born and have no bulk velocity in plasma frame, and second, for obvious reason, we are only interested in the Alfvén waves propagating upward from the transition region toward the corona. Hereafter, the positive wave vector component k_z is used to define that direction.

If we choose to work in the wave frame, there are two advantages: (i) the kinetic energy of each ion is conserved and (ii) it is convenient to express velocity distribution function in terms of two variables (v, μ), where v is the magnitude of ion velocity and $\mu = \cos \theta$, θ being the pitch angle defined in the wave frame. In the following discussion, we shall postulate that thermal velocity spread is small, $\delta v \ll v_A$, and we consider that ions are cold. In the following, we shall denote the distribution function of the s species ions by $F_s(v, \mu)$. Since the generated Alfvénic fluctuations are propagating away from the heavy ions, the pitch-angle distribution $f_s(\mu)$ peaks at $\mu = -1$. We also consider that the velocity dispersion is small so that we may approximately write

$$F_s(v, \mu) = \frac{\delta(v - v_A)}{4\pi v_A^2} f_s(\mu), \quad (5)$$

where f_s is the reduced distribution function defined by

$$f_s \equiv 2\pi \int_0^\infty dv v^2 F_s(v, \mu). \quad (6)$$

The essence of the theory presented in Ref. 23 can be boiled down two kinetic equations: one for the spectral energy of the wave field $W_k^{(s)}$ and the other that deals with the reduced distribution function f_s . These kinetic equations are given below:^{22,23}

$$\begin{aligned} \frac{\partial W_k^{(s)}}{\partial t} &= \frac{\pi \Omega_p^2 \omega_s^2}{4 \omega_p^2 \Omega_s} \int_{-1}^1 d\mu (1 - \mu^2) \delta \left(1 - \frac{|k_z \mu| v_A}{\Omega_s} \right) \\ &\quad \times \left(\frac{m_s v_A^2}{2} f_s + W_k^{(s)} \frac{\partial f_s}{\partial \mu} \right), \quad (7) \\ \frac{\partial f_s}{\partial t} &= \frac{2\pi}{B_0^2} \int d^3 k \frac{\partial}{\partial \mu} (1 - \mu^2) \frac{\partial f_s}{\partial \mu} \left(\frac{\partial W_k}{\partial t} \right), \end{aligned}$$

where s denotes the heavy ion species and p denotes physical quantity associated with the protons; v_A is Alfvén speed; B_0 is the ambient magnetic field intensity; $\Omega_s = e_s B_0 / m_s c$ is the gyro-frequency of the ion species s ; $\omega_s = (4\pi n_s e_s^2 / m_s)^{1/2}$ is the plasma frequency defined with respect to the ion species s ; and k_z is the component of wave vector \mathbf{k} parallel to the ambient magnetic field. To make the discussion self-contained, the derivation of Eq. (7) is outlined in the Appendix.

We are not interested in solving the above kinetic equations explicitly. Important and interesting basic physics can be deduced from these equations without explicit solutions.

On the basis of the wave equation in (7)—the first equation—we see that the first term on the right hand side, which is independent of $W_k^{(s)}$ describes the *spontaneous generation* of Alfvén waves, while the second term describes the *induced absorption* process, which is proportional to the product of $W_k^{(s)}$ and $\partial f_s(\mu)/\partial\mu$. The following condition determines a saturation spectral energy $W_k^{(s)}$:

$$\frac{m_s v_A^2}{2} f_s + W_k^{(s)} \frac{\partial f_s}{\partial \mu} \rightarrow 0, \quad (8)$$

provided the distribution function f_s is known. The particle kinetic equation in (7)—the second equation—contains two additional terms which are proportional to¹⁴

$$\delta(\Omega_s - |k_z \mu| v_A) \left(\frac{m_s v_A^2}{2} f_s + W_k^{(s)} \frac{\partial f_s}{\partial \mu} \right).$$

Since we are only interested in the stationary solution as dictated by Eq. (8), the additional term identically vanishes, and it is sufficient to discuss the stationary solution to the particle kinetic equation, as shown in the second equation in (7). Following the approach taken in Ref. 22, we obtain from the particle kinetic equation in (7), the following explicit solution:

$$f_s(\mu) = \frac{2B_0^2}{\delta B_w^2} \exp\left(-\frac{B_0^2}{\delta B_w^2} (1 - \mu^2)\right), \quad (9)$$

so that $\partial f_s / \partial \mu \approx -2(B_0^2 / \delta B_w^2) f_s$ for $\mu \approx -1$. From Eqs. (8) and (9), we find the spectral wave energy at equilibrium

$$W_k^{(s)} \simeq \frac{\delta B_w^2 m_s v_A^2}{B_0^2 4}. \quad (10)$$

B. Reinterpretation of the ion distribution function $f_s(\mu)$

To clarify the physical meaning of the distribution function $f_s(\mu)$ expressed in Eq. (9), we make use of the saturation spectral energy, Eq. (10), and the relation $(1 - \mu^2)v_A^2 \simeq v_\perp^2$ so that the distribution function f_s may be rewritten as

$$\begin{aligned} f_s(\mu) \rightarrow f_s(v_\perp) &= \frac{m_s}{T_{s\perp}} \exp\left(-\frac{m_s v_A^2}{2T_{s\perp}} (1 - \mu^2)\right) \\ &\approx \frac{m_s}{T_{s\perp}} \exp\left(-\frac{m_s v_\perp^2}{2T_{s\perp}}\right), \end{aligned} \quad (11)$$

where we have introduced a spectral temperature $T_{s\perp}$ such that

$$T_{s\perp} = 2W_k^{(s)} \quad (12)$$

for species s . Coincidentally, Eq. (12) resembles the relation of the brightness temperature associated with the spectral energy of the radiation field in classical theory of radiation.

As stated in Sec. III, in the proposed scenario, the ambient protons are heated by the intrinsic Alfvénic turbulence, which is generated by the heavy ions and other preexisting

Alfvén waves in the region of interest. It is implicitly assumed that the total wave energy density $\delta B_w^2 / 8\pi$ is much higher than the thermal energy density of all these ion species. In other words, it is supposed that

$$\sum_{s \neq p, h} n_s T_{s\perp} \ll \frac{\delta B_w^2}{8\pi} \simeq n_p T_p < n_p T_{p\perp}. \quad (13)$$

The results shown in Eqs. (10) and (12) are consistent with the above supposition made in the scenario since it is assumed that

$$\frac{n_s m_s}{n_p m_p} \ll 1.$$

At this point, we remark that implicit in Eq. (13) is that the time for isotropization of the proton temperature by collisions is effective and short.

V. TEMPERATURE PROFILE IN THE TRANSITION REGION AND LOW CORONA

The central issue of the solar corona is how and why the coronal plasma can reach a temperature as high as two million degrees. The steep temperature gradient in the transition region often creates an impression that an efficient heating process is going on there. It is also puzzling why the heating reaches gradually a saturation point in the low corona. Many scientists have discussed the observed phenomenon on the basis of fluid-theoretical approaches: Although numerous hypothetical models are used to describe possible dissipation processes that might result in heating, the kinetic physics of the process is only conjectured and not theoretically shown.

In this paper, the heating process is discussed on the notion that the turbulent waves can induce microscopic motion of ions so that the ion velocities are randomized and the ion thermal energy is enhanced.^{21,22} The heating process is discussed in terms of the ion velocity distribution function and wave energy density of the Alfvén waves rather than in terms of some model-dependent thermal conductivity or heating rate. As mentioned in Sec. I, we are mainly interested in the low-altitude region of the solar atmosphere shown in Fig. 1 in which the transition region is of particular interest. The kinetic process discussed in the theory has nothing to do with dissipation.

In the following, let us focus our attention to the transition region. For many years, it was conceived as a one-dimensional structure. However, since the *Skylab* mission, the perception is basically changed. Nowadays, it is understood that the chromospheric magnetic field plays a pivotal role. The model proposed by Gabriel³¹ has attracted much interest.^{32,33} This model is compatible with the network/supergranule cells in the chromosphere and also explains the rapid decrease in plasma density with altitude in the region shown in Fig. 1.

For simplicity, let us assume that $T_0 \ll T_{p\perp}$ so that we can write Eq. (4) as

$$n_p T_{p\perp} = \frac{\delta B_w^2}{8\pi}. \quad (14)$$

It is convenient to consider the region close to the central line in the network cell of the Gabriel model, as considered in other theoretical studies, so that the physical situation becomes one dimensional. If we assume that the level of Alfvénic turbulence is roughly the same in the region of interest, then Eq. (14) implies that the thermal energy density is approximately constant as altitude varies. Hence, we may write

$$n_p(z)T_p(z) = n_p(z_0)T_p(z_0), \quad (15)$$

where z_0 is a reference altitude, where the local temperature is $T_p(z_0)$. We note that Eq. (14) is independent of the ambient magnetic field. The only parametric quantity of importance is the wave energy density. We concede that the above approximate explanation may be crude at high altitudes where the level of turbulence may decrease significantly for three reasons: First, the abundance of each species of heavy ions drops quickly; second, Alfvén waves generated by spontaneous process in the transition region or the low corona cannot escape to the solar wind due to ion-cyclotron resonance; and third, at high altitudes, particles gain higher bulk speed so that our theory of heating is no longer meaningful. To test such a conjecture, we carry out numerical calculation on the basis of Eq. (15) and the density model discussed Ref. 34. The calculated proton temperature $T_p(z)$ is shown in Fig. 2 in which the upper panel displays the density profile taken from Ref. 40 and the lower panel describes the temperature profile $T_p(z)$ where we choose to consider $z_0 \simeq 2 \times 10^3$ km, and two initial values 5×10^4 K and 10^5 K. It is seen that the numerical results are qualitatively consistent with observations at low altitudes, say, below 10^4 km. However, it is shown that at high altitudes, the results tend to overestimate, as expected.

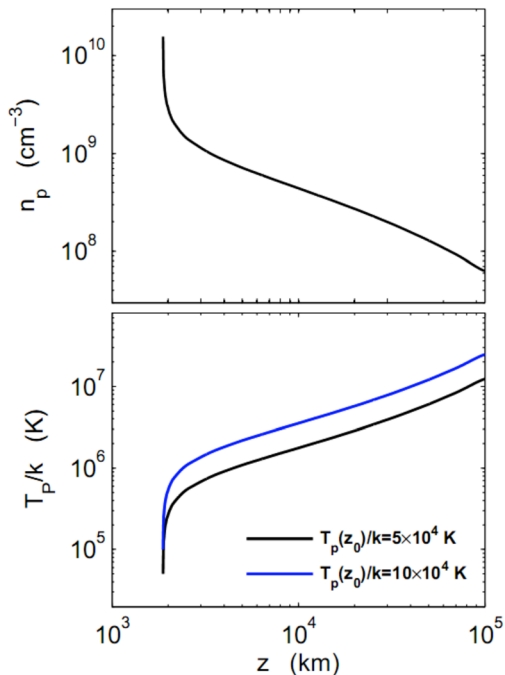


FIG. 2. Top panel: altitude dependence of proton number density according to the density model in Ref. 26. Bottom panel: The proton temperature profiles calculated based on the given density model. We have considered two values of $T_p(z_0)$ for illustration purpose.

Here, we suggest that one may use the present theory to estimate the level of Alfvénic turbulence if the peak proton temperature and the Alfvén speed are known from numerical models. For example, let us rewrite Eq. (14) to express it in the following form:

$$T_p \approx \frac{\delta B_w^2}{B_0^2} \frac{m_p v_A^2}{2}. \quad (16)$$

According to Ref. 22, where the Alfvén speed is around $(1 \sim 2) \times 10^3$ km/s, the peak proton temperature is $100 \sim 200$ eV. It implies that in the low corona, $10^{-3} < \delta B_w^2 / B_0^2 < 10^{-2}$.

VI. DISCUSSION AND CONCLUSIONS

The proposed theory considers that the sun is quiet such that the corona may be supposed to be spherically symmetric. The primary objective is to explain the underlying physics responsible for the observed heating in the low altitude region, as described in Fig. 1. The proposed scenario is summarized as follows: First of all, we show that the heavy minor ions, which are continuously created in the transition region due to collisional ionization by hot electrons, can generate enhanced Alfvénic fluctuations. These broadband Alfvén waves in general have steady-state spectral energy because of the nature of spontaneous generation process.²³ These waves together with other randomly excited intrinsic Alfvén waves due to local disturbances and energetic particles in the solar plasma can result in induced heating of the ambient protons which can in turn heat electrons via collisions. The hot electrons can then produce the minor ions. All these processes are operative continuously in time and globally in space. Of course, the scenario has made use of a necessary assumption: neutral atoms are continuously replenished from the photosphere. Many minor ions are expected to be carried away by the Alfvén waves.

The present discussion is mainly interested in the heating process occurring in the transition region and the low corona. Acceleration of the plasma is not studied. However, we may digress briefly how the solar wind is originated within the context of the proposed scenario. We point out that while the heating process, which occurs mainly in the perpendicular direction, is in progress, the protons are gradually picked up by the propagating Alfvénic turbulence. Through this acceleration process, the ambient protons attain a finite bulk velocity along the open magnetic field lines so that eventually the solar wind is formed.

In the following, we remark that the proposed theory is in line with related observations. First of all, according to Refs. 21 and 22, heating of protons due to Alfvénic turbulence in general would lead to temperature anisotropy such that $T_{p\perp} > T_{p\parallel}$. This is consistent with observed proton temperature. Second, in the present theory we see that according to Eq. (11), under the same level of turbulence, the minor ions can attain a higher perpendicular temperature such that

$$T_{s\perp} = \frac{m_s}{m_p} T_{p\perp}. \quad (17)$$

This finding is in consistent with the SOHO observations reported in Refs. 35–38 that for a heavy ion species, the perpendicular temperature is greater than the mass ratio. It is known in plasma physics that the heavier the mass the longer the relaxation time of isotropization. This is why the observed proton anisotropy is usually smaller than that of the heavy ion species. In solar physics, it is commonly assumed that electrons and protons in the hot corona have same temperature. However, the heating process discussed in this paper is only effective for ions. It is supposed in our discussion that the electrons are heated by the hot protons by collisions and eventually reach thermal equilibrium. Because in general, this process requires a time lag, the electron temperature is expected to be somewhat lower than that of protons. This expectation is consistent with the general observations of the high corona and coronal holes.³⁹

Here, we also discuss several subtle points of the proposed theory. First, although in the discussion of the spontaneous generation of Alfvénic fluctuations we emphasize heavy minor ions, other preexisting Alfvén waves in the atmosphere are also implicitly involved. This point is seen from the quantity δB_w^2 , which affects the spectral energy $W_k^{(s)}$ and the spectral temperature defined in Eq. (10) and Eq. (12). A far reaching implication is that the heavy ions can convert other intrinsic Alfvén waves to Alfvénic fluctuations which is effective for the heating process. An essential requirement of the quasilinear theory is that the waves have a broad spectrum. That is why we are interested in the minor ions. Second, we point out that the saturation level of the wave spectral energy $W_k^{(s)}$ associated with a given ion species s , which is discussed in Sec. IV A, is independent of the abundance of the ion species. Third, in view of the fact that the abundance of each species of heavy ions is very small, one may wonder whether these ions can really play important and significant roles. One important quantity that deserves discussion is the transient time of the generation process. To discuss this point, let us return to the first equation in (7) and ignore the absorption term so that we obtain

$$\frac{\partial W_k^{(s)}}{\partial t} = \frac{\pi}{2} \left(\frac{e_s n_s m_s}{e n_p m_p} \right) \Omega_p (1 - \mu_0^2) \left(\frac{B_0^2}{\delta B_w^2} \right) m_s v_A^2, \quad (18)$$

where $|\mu_0| = |\Omega_s/k_z v_A| \approx 1$. From Eq. (9), we consider $(1 - \mu_0^2)(B_0^2/\delta B_w^2) \approx \mathcal{O}(1)$. If we denote the time scale for the spectral energy to reach the saturation level by τ , then we find

$$\tau \approx \left\{ \frac{e_s n_s m_s}{e n_p m_p} \Omega_p \left(\frac{B_0^2}{\delta B_w^2} \right) \right\}^{-1}. \quad (19)$$

In the present discussion, the magnitude of the ratio $\delta B_w^2/B_0^2$ is considered as a free parameter. At low altitudes, there is no “typical magnetic field.” Whatever value we shall use is hypothetical. However, it is reasonable to consider that for heavy ions with high charge state, the transient time is in general much shorter than the proton gyro period. If we consider that the local proton gyro-frequency is in the range 1 kHz to 10² kHz and assume

$$\frac{e_s n_s m_s}{e n_p m_p} \left(\frac{B_0^2}{\delta B_w^2} \right) \sim \mathcal{O}(1),$$

then the transient time is of order Ω_p^{-1} , which is indeed very short.

The proposed theory leads to two major conclusions. (i) Although the temperature in the corona appears much higher than that in the low transition region, the thermal energy density of the plasma is approximately conserved. The steep temperature gradient often misleads scientists to think that some kind of mechanical energy is converted to thermal energy. (ii) The minor ions may play a pivotal role in producing the hot corona. The key to the longstanding issue is that the ion temperature in the corona needs reinterpretation.

Finally we remark that it is regrettable that the Alfvénic fluctuations in the proposed theory cannot be identified by available methods of observation because these superposing waves have random phases and broadband frequencies. Moreover, these Alfvén waves cannot escape from the corona and therefore they cannot be seen in the solar wind.

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APPENDIX: DERIVATION OF WAVE KINETIC EQUATION

We first introduce the Fourier-Laplace transform of a fluctuating quantity $\delta A = \delta A(t, \mathbf{r})$,

$$\delta A(\omega, \mathbf{k}) = \lim_{\Delta \rightarrow 0} \int d^3r \int_0^\infty dt \exp(i\omega t - \Delta t - i\mathbf{k} \cdot \mathbf{r}) \delta A(t, \mathbf{r}). \quad (A1)$$

If we adopt the conventional coordinate system in which the wave vector and the ambient magnetic field are designated as $\mathbf{k} = (k_\perp, 0, k_z)$ and $\mathbf{B}_0 = B_0 \hat{\mathbf{e}}_z$, then it is known in plasma physics that the Alfvén mode has wave electric field and perturbation current in the direction parallel to the x axis^{40,41} so that from the field equation, we obtain the dispersion equation

$$\begin{aligned} \Lambda_{xx}(\omega, \mathbf{k}) &\equiv [-c^2 k_z^2 + \omega^2 \epsilon_{xx}(\omega, \mathbf{k})] \delta E_x(\omega, \mathbf{k}) \\ &= -4\pi\omega i \sum_s \delta j_{sx}(\omega, \mathbf{k}), \end{aligned} \quad (A2)$$

where ϵ_{xx} is the xx component of dielectric tensor; and $j_{sx}(\omega, \mathbf{k})$ denotes the Fourier-Laplace transform of the fluctuating current of s species ions in x direction,

$$\delta j_{sx} = e_s \int d^3v v_x \delta N_s(\mathbf{v}, \omega, \mathbf{k}), \quad (\text{A3})$$

and $\delta N_s(\mathbf{v}, \omega, \mathbf{k})$ is the Klimontovich density function.⁴² We write $\epsilon_{xx} = \text{Re}\epsilon_{xx} + i\text{Im}\epsilon_{xx}$, where Re and Im denote the real and imaginary part and assume $\text{Re}\epsilon_{xx} \gg \text{Im}\epsilon_{xx}$. If we consider that the real frequency of the wave is not affected by the fluctuating current and $\text{Im}\epsilon_{xx}$, then we can obtain the approximate dispersion relation

$$[-c^2k_z^2 + \omega_k^2 \text{Re}\epsilon_{xx}(\omega_k, \mathbf{k})] = 0, \quad (\text{A4})$$

which yields $\omega = \pm\omega_k$, the real frequency ω_k of the Alfvén mode. When the ambient plasma is cold, we have

$$\text{Re}\epsilon_{xx} \simeq 1 + \frac{\omega_p^2}{\Omega_p^2} \simeq \frac{\omega_p^2}{\Omega_p^2} \gg 1. \quad (\text{A5})$$

Consequently, we can write

$$\text{Im}\epsilon_{xx} = -\pi \sum_s \int d^3v \left(\frac{n^2}{b_s^2} \right) J_n^2(b_s) \hat{R} F_s \delta(\omega_k - n\Omega_s - k_z v_z), \quad (\text{A6})$$

where $b_s = k_\perp v_\perp / \Omega_s$,

$$\hat{R} \equiv \frac{\partial}{\partial v_\perp} - \frac{k_z v_z}{\omega} \left(\frac{\partial}{\partial v_\perp} - \frac{v_\perp}{v_z} \frac{\partial}{\partial v_z} \right),$$

and the definitions for other quantities are given in the text. We now discuss the effect of the fluctuating current. If we postulate that Eq. (A2) eventually supports a complex root, which has a small positive imaginary part in the complex ω plane so that we can approximate Eq. (A2) by

$$\left[i\omega^2 \text{Im}\epsilon_{xx}(\omega, \mathbf{k}) + (\omega - \omega_k + i\gamma_k) \frac{\partial}{\partial \omega_k} (\omega_k^2 \text{Re}\epsilon_{xx}(\omega_k, \mathbf{k})) \right] \times \delta E_x(\omega, \mathbf{k}) = -4\pi\omega i \sum_s \delta j_{sx}(\omega, \mathbf{k}), \quad (\text{A7})$$

and we obtain

$$\begin{aligned} & \gamma_k \frac{\partial}{\partial \omega_k} (\omega_k^2 \text{Re}\epsilon_{xx}(\omega_k, \mathbf{k})) \delta E_x(\omega_k, \mathbf{k}) \\ &= -\omega_k^2 \text{Im}\epsilon_{xx}(\omega_k, \mathbf{k}) \delta E_x(\omega_k, \mathbf{k}) - 4\pi\omega_k \sum_s \delta j_{sx}(\omega_k, \mathbf{k}). \end{aligned} \quad (\text{A8})$$

In deriving Eq. (A8), we have made use of the Landau convention

$$\lim_{\Delta \rightarrow 0} \frac{1}{\omega - \omega_k + i\Delta} = -i\pi \delta(\omega - \omega_k).$$

Multiplying Eq. (A8) by the complex conjugate $\delta E_x^*(\omega_k, \mathbf{k})$, symmetrizing and taking ensemble average $\langle \rangle$ of all terms, we then obtain

$$\begin{aligned} & \frac{\partial}{\partial \omega_k} (\omega_k^2 \text{Re}\epsilon_{xx}(\omega_k, \mathbf{k})) \frac{\partial}{\partial t} \langle \delta E_x^2(\omega_k, \mathbf{k}) \rangle \\ &= -2\omega_k^2 \text{Im}\epsilon_{xx}(\omega_k, \mathbf{k}) \langle \delta E_x^2(\omega_k, \mathbf{k}) \rangle \\ &+ 8\pi^3 \omega_k \sum_s \langle \delta j_{sx}(\omega_k, \mathbf{k}) \delta j_{sx}^*(\omega_k, \mathbf{k}) \rangle, \end{aligned} \quad (\text{A9})$$

where we have written

$$2\gamma_k \langle \delta E_x^2 \rangle = \frac{\partial}{\partial t} \langle \delta E_x^2 \rangle,$$

and have made use of the following approximate expression:

$$\delta E_x(\omega, \mathbf{k}) = -4\pi^2 \omega \frac{\delta(\omega - \omega_k) \delta j_{sx}(\omega_k, \mathbf{k})}{\frac{\partial}{\partial \omega_k} (\omega_k^2 \text{Re}\epsilon_{xx}(\omega_k, \mathbf{k}))}. \quad (\text{A10})$$

In obtaining Eq. (A10), we have made use of an iterative approximation by considering

$$\omega^2 \text{Im}\epsilon_{xx}(\omega, \mathbf{k}) + \gamma_k \frac{\partial}{\partial \omega_k} (\omega_k^2 \text{Re}\epsilon_{xx}(\omega_k, \mathbf{k})) \approx 0 \quad (\text{A11})$$

because conceptually Eq. (A11) is derivable based on Vlasov theory. As a result, we have

$$\begin{aligned} & \lim_{\Delta \rightarrow 0} (\omega - \omega_k + i\Delta) \frac{\partial}{\partial \omega_k} (\omega_k^2 \text{Re}\epsilon_{xx}(\omega_k, \mathbf{k})) \delta E_x(\omega_k, \mathbf{k}) \\ &= -4\pi\omega i \sum_s \delta j_{sx}(\omega, \mathbf{k}), \end{aligned}$$

and consequently, we obtain

$$\begin{aligned} \frac{\partial}{\partial t} \langle |\delta E_x(\omega, \mathbf{k})|^2 \rangle &= -2\Gamma_k \langle |\delta E_x(\omega, \mathbf{k})|^2 \rangle \\ &+ 32\pi^3 \omega_k^2 \sum_s \frac{\langle \delta j_{sx}(\omega_k, \mathbf{k}) \delta j_{sx}^*(\omega_k, \mathbf{k}) \rangle}{\left[\frac{\partial}{\partial \omega_k} (\omega_k^2 \text{Re}\epsilon_{xx}(\omega_k, \mathbf{k})) \right]^2} \end{aligned} \quad (\text{A12})$$

and is Γ_k the induced absorption coefficient which is supposed to be negative, meaning absorption

$$2\Gamma_k(\omega_k, \mathbf{k}) \equiv \frac{\omega_k^2 \text{Im}\epsilon_{xx}(\omega_k, \mathbf{k})}{\frac{\partial}{\partial \omega_k} (\omega_k^2 \text{Re}\epsilon_{xx}(\omega_k, \mathbf{k}))}. \quad (\text{A13})$$

Equation (A12) describes the quantity $\langle |\delta E_x(\omega, \mathbf{k})|^2 \rangle = \langle \delta E_x(\omega, \mathbf{k}) \delta E_x^*(\omega, \mathbf{k}) \rangle$, where $\delta E_x(\omega, \mathbf{k})$ is the Fourier-Laplace transform of the fluctuating wave electric field. From observational point of view, it is desirable to express Eq. (A12) in terms of the Fourier transform of the correlation function $\langle \delta E_x(\mathbf{r}, t) \delta E_x(\mathbf{r} - \mathbf{r}', t - t') \rangle$ at a given frequency. Making use of the method by Klimontovich,⁴² one may write

$$\langle A(\mathbf{k}, \omega) B^*(\mathbf{k}, \omega) \rangle = \lim_{\Delta \rightarrow 0} \frac{1}{2\Delta V} \langle AB \rangle_{\mathbf{k}, \omega}, \quad (\text{A14})$$

where V denotes the volume of the system.

For Alfvén waves, we have

$$\frac{\partial}{\partial \omega_k} (\omega_k^2 \text{Re}\epsilon_{xx}) = 2\omega_k \left(\frac{\omega_p^2}{\Omega_p^2} \right). \quad (\text{A15})$$

Thus, we obtain from Eq. (A12)

$$\begin{aligned} \frac{\partial}{\partial t} W_k &= -2\Gamma_k W_k + 2\pi^2 \sum_s \lim_{\Delta \rightarrow 0} \left(\frac{\Omega_p^2}{\omega_p^2} \right) \frac{\Delta}{V} \\ &\times \langle \delta j_{sx}(\omega_k, \mathbf{k}) \delta j_{sx}^*(\omega_k, \mathbf{k}) \rangle. \end{aligned} \quad (\text{A16})$$

To calculate $\langle \delta j_{sx}(\omega_{\mathbf{k}}, \mathbf{k}) \delta j_{sx}^*(\omega_{\mathbf{k}}, \mathbf{k}) \rangle$ where $\delta j_{sx}(\omega, \mathbf{k}) = e_s \int d^3 v v_x \delta N_s(\mathbf{v}, \omega, \mathbf{k})$, we first discuss $\delta N_s(\mathbf{v}, \omega, \mathbf{k})$. Let us consider that $\delta N_s(\mathbf{v}, \omega, \mathbf{k})$ satisfies the following equation:

$$\left\{ \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla + \left(\frac{e_s}{m_s c} \mathbf{v} \times \mathbf{B}_0 \right) \cdot \frac{\partial}{\partial \mathbf{v}} \right\} \delta N_s(t, \mathbf{v}, \mathbf{r}) = 0, \quad (\text{A17})$$

without considering the effect of self-consistent field. According to the definition given in Eq. (1), this equation leads to

$$\lim_{\Delta \rightarrow 0} \left\{ i(\omega - \mathbf{k} \cdot \mathbf{v} + i\Delta) + \Omega_s \frac{\partial}{\partial \varphi} \right\} \delta N_s(\omega, \mathbf{v}, \mathbf{k}) = \delta N_s(0, \mathbf{v}, \mathbf{k}), \quad (\text{A18})$$

where φ is the gyro phase angle, $\varphi = \cos^{-1}(v_x/v_\perp)$. Standard method of solution leads to

$$\begin{aligned} \delta N_s(\omega, \mathbf{v}, \mathbf{k}) &= -i \sum_n \frac{n J_n(b_s)}{b_s} \frac{\exp(-in\varphi)}{(\omega - n\Omega_s - k_z v_z + i\Delta)} \delta N_s(0, \mathbf{v}), \\ \delta N_s^*(\omega, \mathbf{v}', \mathbf{k}) &= i \sum_m \frac{m J_m(b_s)}{b_s} \frac{\exp(im\varphi)}{(\omega - m\Omega_s - k_z v_z - i\Delta)} \delta N_s(0, \mathbf{v}'). \end{aligned} \quad (\text{A19})$$

Making use of the following identity due to self-correlation⁴²

$$\langle \delta N_s(\mathbf{v}) \delta N_s(\mathbf{v}') \rangle = n_s F_s(\mathbf{v}) \delta(\mathbf{v} - \mathbf{v}'),$$

and Eq. (A19) one finds

$$\begin{aligned} \lim_{\Delta \rightarrow 0} \Delta e^2 \int d^3 v v_x \int d^3 v' v'_x \langle \delta N_s(\mathbf{v}, \omega, \mathbf{k}) \delta N_s^*(\mathbf{v}', \omega, \mathbf{k}) \rangle \\ = \lim_{\Delta \rightarrow 0} \sum_{n=-\infty}^{\infty} \left(\frac{nv_\perp}{b_s} J_n(b_s) \right)^2 \\ \times \frac{V \Delta n_s e_s^2 F_s(\mathbf{v}(t))}{(\omega - k_z v_z - n\Omega_s + i\Delta)(\omega - k_z v_z - n\Omega_s - i\Delta)} \\ = \sum_{n=-\infty}^{\infty} \left(\frac{nv_\perp}{b_s} J_n(b_s) \right)^2 \pi n_s e_s^2 V F_s(\mathbf{v}(t)) \delta(\omega - k_z v_z - n\Omega_s). \end{aligned} \quad (\text{A20})$$

Finally, from Eqs. (A16) and (A20), we obtain

$$\begin{aligned} \frac{\partial W_{\mathbf{k}}}{\partial t} &= -2\Gamma_{\mathbf{k}} W_{\mathbf{k}} + 4\pi^2 m_s \omega_s^2 \sum_s \lim_{\Delta \rightarrow 0} \left(\frac{\Omega_p^2}{\omega_p^2} \right) \\ &\times \sum_{n=-\infty}^{\infty} \left(\frac{nv_\perp}{b_s} J_n(b_s) \right)^2 F_s(\mathbf{v}(t)) \delta(\omega_{\mathbf{k}} - k_z v_z - n\Omega_s). \end{aligned} \quad (\text{A21})$$

To simplify this equation, we consider several points: (a) $b_s \ll 1$, (b) $n = 1$, (c) we choose to work in the wave frame and adopt a spherical coordinate system with two variables (v, μ) where $\mu = \cos \theta$ and θ is the pitch angle defined in the wave frame. We have used the wave spectral energy defined in Eq. (1). If we make use of Eq. (A13), it can be shown that the wave kinetic equation reduces to

$$\begin{aligned} \frac{\partial W_{\mathbf{k}}}{\partial t} &= \frac{\pi^2}{2} \sum_s \int_0^\infty dv v^2 \int_{-1}^1 d\mu \omega_s^2 \left(\frac{\Omega_p^2}{\omega_p^2} \right) \delta(\Omega_s - |\mu k_z| v) \\ &\times \left(\frac{m_s v^2}{2} F_s + W_{\mathbf{k}} \frac{v}{v_A} \frac{\partial F_s}{\partial \mu} \right). \end{aligned} \quad (\text{A22})$$

On the right hand side of Eq. (A22), the first term represents spontaneous generation, while the second term denotes induced absorption.

Note that the present Appendix is dedicated to the derivation of Alfvén wave kinetic equation, including the spontaneous emission term. Customary discussions found in the literature on Alfvén wave quasilinear dynamics often do not include spontaneous effects. See, e.g., Refs. 43 and 44, just to cite two. In this regard, it is worth mentioning that a recent paper generalizes the traditional quasilinear theory to not only include spontaneous thermal effects, but also to extend the analysis to aperiodic modes.⁴⁵ However, Ref. 45 pertains to unmagnetized plasmas, whereas the present Alfvén wave problem requires ambient magnetic field.

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