Dust voids in collision-dominated plasmas with negative ions

Xiang Zheng,[1] Yinhua Chen,[1,a)] Zuquan Hu,[1] Guanwen Wang,[1] Feng Huang,[1] Chuanfei Dong,[1] and M. Y. Yu[2]
[1]*Key Laboratory of Basic Plasma Physics, USTC, Chinese Academy of Sciences and Department of Modern Physics, University of Science and Technology of China, Hefei, 230026 China*
[2]*Institute for Fusion Theory and Simulation, Zhejiang University, 310027 Hangzhou, China*

(Received 5 December 2008; accepted 31 December 2008; published online 12 February 2009)

Using fluid theory, the properties of voids in collision-dominated plasmas containing negative ions are studied. The profiles of the charged-particle densities in the void region are obtained. It is also shown that with an increase of the negative-ion concentration, the electric field, the ion drift velocity, the dust charge at the void edge, as well as the void size decrease. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3073679]

I. INTRODUCTION

Much attention has been paid to plasmas containing dust grains. Phenomena investigated include dust waves, dust solitons and shocks, dust transport, dust lattices, dust clusters, dust voids, dust balls, light scattering by and chemical evolution of dust grains.[1–15] Charged micron or submicron particulates are also important in many terrestrial applications and space plasmas, including electromagnetic wave scattering,[16] chemical evolution of stars and planets,[17] as well as processing of microelectronic devices and in plasma-related nanotechnology.[18] However, several fundamental problems, including the dust charging process,[19,20] the effective dust-dust interaction force,[21] and the dusty sheath,[22] etc., are still not fully understood.

Voids in a dusty plasma are regions in which no dust exists. They normally have a sharp boundary, which can oscillate at several Hz, within the dust-acoustic time scale.[23] The size of a void is typically of the order of the ion-neutral collision mean free path. Several mechanisms for void formation have been proposed. Tsytovich *et al.*[24] recognized that the dust grains can easily absorb electrons and thereby increase the ionization rate. Regions with positive electric potential with respect to the surrounding dusty plasma are formed, causing the ions in them to move outward. The highly charged dust grains experience both the drag force exerted by the outward moving ions and the inward electrostatic force. In regions where the ion drag force exceeds the electric force, the dust grains are pushed out. Prabhuram *et al.*[3] proposed that dust grains can also be driven out by ionization waves. Morfill *et al.*[25] found that the plasma pressure can play an important role in the formation of voids. Other authors have investigated the stability of dust voids and their nonlinear evolution.[26,27]

Electronegative plasmas are widely used in modern technological applications.[13,28–30] Recently, plasmas containing both negative ions and dust grains have been receiving much attention after Vyas and Kushner[28] found that at high substrate bias, voids in a dusty plasma tend to disappear upon the addition of electronegative gas. Denysenko and Yu[29] considered the effect of negative-ion drag on the dust grains. Gan *et al.*[30] found that negative ions can significantly affect the stationary state and characteristics of the waves in the sheath region of a radio-frequency discharge.

In this paper we investigate the properties of dust voids in collision-dominated plasmas, where ion-neutral collisional friction can be significant.[13,29,31] The analytical model proposed in Ref. 26 for studying dust voids in collisional plasmas is extended to account for the effect of negative ions. Invoking charge and force balance at the void boundary, we obtain the void size and the dust charge at the void edge. The effects of ion-neutral collisions and concentration of negative ions on the spatial profiles of the particle densities, the electric field, and the ion velocity for different negative ion densities are investigated.

II. ONE-DIMENSIONAL MODEL OF STEADY VOIDS

We shall follow the practical normalization used in Ref. 26. Accordingly, the ion, electron, and negative-ion densities n_i, n_e, and n_- are normalized by the ion density n_{i0} at the void center. The electric field E, the distance x, and the velocity u of the ion flow are normalized by $aT_e\sqrt{\tau}/ed_i^2E$, $d_i^2/a\sqrt{\tau}x$, and $\sqrt{2T_i/m_i}$, respectively, where $d_i=(T_i/4\pi n_{0i}e^2)^{1/2}$ is the ion Debye length, a is the radius of the dust grain, and $\tau=T_i/T_e$.

For simplicity, we consider a one-dimensional model of the void in an electronegative dusty plasma. The void center is at $x=0$, and the boundaries are at $x=\pm x_v$. The electrons and the negative ions are assumed to obey the Boltzmann density profile[32]

$$n_e=n_{0e}\exp(\phi), \tag{1}$$

$$n_-=n_{0-}\exp(\phi/\tau_-), \tag{2}$$

where $\tau_-=T_-/T_e$. From the steady-state momentum equation, we obtain for the positive ions,[26]

$$\frac{\partial u}{\partial x}=-\frac{2}{x_n}(2+\alpha_u u)+E, \tag{3}$$

where the ion thermal pressure is neglected. The first term on the right-hand side is from ion-neutral collisions, with x_n and

[a)]Electronic mail: yhchen@ustc.edu.cn.

α_u/x_n related to the ion mobility and ion-neutral collision mean free path, respectively. The positive ion continuity equation including ionization effects can then be written as[26]

$$\frac{\partial\psi}{\partial x}=\frac{\exp(\phi)+\epsilon\exp(\phi/\tau_-)}{x_{0i}}, \quad (4)$$

where $\psi=nu$ is the ion flux, $\epsilon=n_{-0}/n_{i0}$ is the concentration of the negative ions, and $x_{0i}=n_{0i}av_{Ti}\sqrt{2T_i/m_i}/n_{0e}\nu_i d_i^2$ is the normalized ionization mean free path. The Poisson equation

$$\frac{\partial E}{\partial x}=\left(\frac{d_i}{a}\right)^2(n-n_e-n_-) \quad (5)$$

completes the system of equations for the void region.

From Eqs. (1)–(5), we obtain

$$\frac{dE}{dx}=-\frac{d_i^2}{a^2}\left[(1-\epsilon)\exp(\phi)+\epsilon\exp\left(\frac{\phi}{\tau_-}\right)-\frac{\psi}{u}\right], \quad (6)$$

$$\frac{du}{dx}=-\frac{2}{x_n}(2+\alpha_u u)-\frac{d\phi}{dx}, \quad (7)$$

$$\frac{d\psi}{dx}=\frac{(1-\epsilon)\exp(\phi)+\epsilon\exp(\phi/\tau_-)}{x_{0i}}, \quad (8)$$

where the electrostatic potential ϕ is given by $d\phi/dx=-E$.

The forces acting on the dust grains with charge Z_d at the boundary of the quasineutral void are the electrostatic force $F_E=-Z_d eE$ and the ion drag force $F_{\rm dr}=\alpha_{\rm dr}(u,\tau/z)zn_dZ_du$, where $z=Z_de^2/aT_e$ and the drag coefficient $\alpha_{\rm dr}$ is calculated taking into account both the capture force and the Coulomb scattering force.[21,26] The force balance equation of dust grains at void boundary is then

$$\alpha_{\rm dr}(u,\tau/z)nuz=E, \quad (9)$$

where the drag coefficient $\alpha_{\rm dr}$ is given by[26]

$$\alpha_{\rm dr}=\mathrm{erf}(u)K(u)/u, \quad (10)$$

where $K(u)=(-1+4u^2+4u^4)t-2(1-2u^2)t+4\ln(d_i/a)$ and $t=\tau/z$.

It is also necessary to consider the electron, positive-ion, and negative-ion currents on the dust grains. At the void boundary, the current balance equation is

$$I_e+I_++I_-=0, \quad (11)$$

where the electron, negative-ion, and positive-ion grain-charging currents are[31]

$$I_e=-4\pi a^2 n_{e0}e\left(\frac{T_e}{2\pi m_e}\right)^{1/2}\exp\left(\frac{e\phi}{T_e}\right), \quad (12)$$

$$I_-=-4\pi a^2 n_{-0}e\left(\frac{T_-}{2\pi m_-}\right)^{1/2}\exp\left(\frac{e\phi}{T_-}\right), \quad (13)$$

$$I_+=2\pi a^2 n_{i0}Ze\alpha_{\rm ch}\left(\frac{T_e}{2m_i}\right)^{1/2}. \quad (14)$$

Substituting the grain currents into Eq. (11), one gets the normalized grain-current balance equation

$$n_e\exp(-z)+\sqrt{\tau_-\mu}n_-\exp(-z/\tau_-)=n\sqrt{\pi\mu}z/2\alpha_{\rm ch}, \quad (15)$$

where $\mu=m_e/m_i$ and

$$\alpha_{\rm ch}=\frac{\mathrm{erf}(u)}{8u}(2+t+3u^2t)-\frac{t\exp(-u^2)}{4\sqrt{\pi}} \quad (16)$$

is the charging coefficient.[26]

Substituting Eq. (10) into Eq. (9), we obtain

$$E(x_v)-Z_v\,\mathrm{erf}(u_v)K(u_v)\psi/u_v=0 \quad (17)$$

for the force balance at the void boundary. From Eqs. (15) and (16), we find the corresponding charging equation

$$(1-\epsilon)\exp[-z_v+\phi(x_v)]$$
$$=-\sqrt{\tau_-\mu}\epsilon\exp\left[-\frac{z_v}{\tau_-}+\frac{\phi(x_v)}{\tau_-}\right]$$
$$+z_v\sqrt{\pi\mu}\left[\frac{\mathrm{erf}(u_v)}{8u_v}(2+t+3u_v^2t)-\frac{t\exp(-u_v^2)}{4\sqrt{\pi}}\right]\frac{\psi}{2u_v}, \quad (18)$$

which together with Eq. (17) are the boundary conditions to be satisfied at the void boundary $x=x_v$. At the center of the void $x=0$, symmetry requires $E=0$, and quasineutrality requires $n_e+n_-=n=1$.[24,26]

We now consider the dusty region. The Poisson's equation in this region is

$$\frac{\partial E}{\partial x}=\left(\frac{d_i}{a}\right)^2(n-n_e-n_--Z_dn_d), \quad (19)$$

where Z_dn_d is the dust charge density normalized by n_{i0}. The ion flux equation is

$$\frac{\partial\psi}{\partial x}=-nZ_dn_d\alpha_{\rm ch}. \quad (20)$$

The ion momentum equation can be written as

$$\frac{\partial(u^2+\phi)}{\partial x}=-zuZ_dn_d\alpha_{\rm dr}, \quad (21)$$

where the right-hand side is from the ion drag force. The dust density n_d at the void boundary can be obtained form Eqs. (10) and (19)–(21),

$$n_{dv}=\frac{(1-2u_v^2)W+A}{\left(\frac{\tau d_i^2}{a^2}-\frac{Z_v\psi_v\alpha_{\rm dr}\alpha_{\rm ch}}{u_v}\right)(1-2u_v^2)-B}, \quad (22)$$

where

$$W=\left(\frac{\psi_v}{u_v}-Q\right)\frac{\tau d_i^2}{a^2}+\frac{u_v\psi_v\alpha_{\rm dr}Z_v}{R}\frac{\partial\alpha_{\rm dr}Z}{\partial Z}-Z_vQ\alpha_{\rm dr}\alpha_u,$$

$$Q=(1-\epsilon)\exp[\phi(x_v)]+\epsilon\exp[\phi(x)/\tau], \quad (23)$$

$$R=1+\mathrm{erf}(u_v)/4u_vZ_v\alpha_{\rm ch},$$

and

$$A = \frac{nu}{R}\frac{\partial z\alpha_{\mathrm{dr}}}{\partial z}\left\{ nuz\alpha_{\mathrm{dr}} + \frac{2u\alpha_{\mathrm{ch}}n_e}{n} - \frac{u(2+\alpha_u u)}{x_n} - \frac{\sqrt{\alpha_{\mathrm{ch}}}}{\alpha_{\mathrm{ch}}}\frac{\partial\alpha_{\mathrm{ch}}}{\partial u}\left[u\psi z\alpha_{\mathrm{dr}} - \alpha_{\mathrm{ch}}\frac{u}{\psi} - \frac{u^2}{x_n}(2+\alpha_u)\right]\right\} + z\frac{\partial \alpha_{\mathrm{dr}}}{\partial z}\left[\frac{u^2 z\psi\alpha_{\mathrm{dr}}}{\tau} - \frac{u^2}{x_n}(2+\alpha_u) - \alpha_{\mathrm{ch}}\frac{u}{\psi}\right]\psi, \tag{24}$$

$$B = \frac{nu}{R}\frac{\partial z\alpha_{\mathrm{dr}}}{\partial z}\left[u\left(2\alpha_{\mathrm{ch}} - \frac{z}{\tau}\alpha_{\mathrm{dr}}\right) - \frac{\partial\alpha_{\mathrm{ch}}}{\partial z} + \frac{u^2 z\alpha_{\mathrm{dr}}}{\tau\alpha_{\mathrm{ch}}}\frac{\partial\alpha_{\mathrm{ch}}}{\partial u}\right] + \psi z\left(\alpha_{\mathrm{ch}} - \frac{u^2\alpha_{\mathrm{dr}}z}{\tau}\right)\frac{\partial\alpha_{\mathrm{dr}}}{\partial u}, \tag{25}$$

where we have assumed *a priori* that the void boundary is sharp, as observed in the experiments.[5,33]

III. NUMERICAL RESULTS

For concreteness, we set $d_i=0.001$, $a=0.01$, $T_i/T_e=0.05$, $m_e/m_i=5.4\times10^{-4}$, $T_-/T_e=0.05$, which are typical for low-temperature laboratory discharges.[20,28] We also assume that the normalized ionization mean free path is $x_{0i}=6.67$. Using Eqs. (17), (18), and (22) to determine if the void can exist, as well as the location of the void boundary, we then solve Eqs. (6)–(8) numerically.

In Fig. 1, the profiles of the charged-particle densities in the void are shown for $x_n=50$ and negative-ion concentrations $\epsilon=0,0.4,0.8$. The curves are terminated at the corresponding void edges, namely, at $x_v=14.884, 14.159, 11.426$, respectively. The typical void size is consistent with that of Vyas and Kushner[28] and Denysenko *et al.*[29] We also see that the charged-particle density profiles depend strongly on the negative-ion concentration. As the latter increases, both the positive and negative ion densities decrease rapidly towards the void boundary, but the electron density (already reduced due to the increased negative ions) decreases slowly. Furthermore, the negative ions tend to concentrate at the center of the void.

Figure 2 shows the profiles of (a) the electric field E and (b) the positive ion drift velocity u for the same plasma conditions as in Fig. 1. Within the void the electric field increases with x. We also see that an increase of the negative-ion concentration tends to lower the electric field in the quasineutral void. This can be attributed to the fact that in the latter, a larger negative-ion density corresponds to a much lower electron density, but the much heavier, fewer, and slower (colder) negative ions obviously cannot take over the role of the electrons they replace in the ionization process. Thus, in the presence of negative ions, the ionization rate will be lower and the electric field in the void smaller. The ions in the void are driven by the electric field. Accordingly, we can see that the positive ion velocity u increases towards the edge of the void. For the same reason the negative-ion velocity decreases as their density increases. As expected, the ion drift velocity depends primarily on the electric field.

Figure 3(a) shows the relation between the negative-ion density and the void size for different ion mean free paths x_n. One can see that an increase of x_n leads to a decrease of the

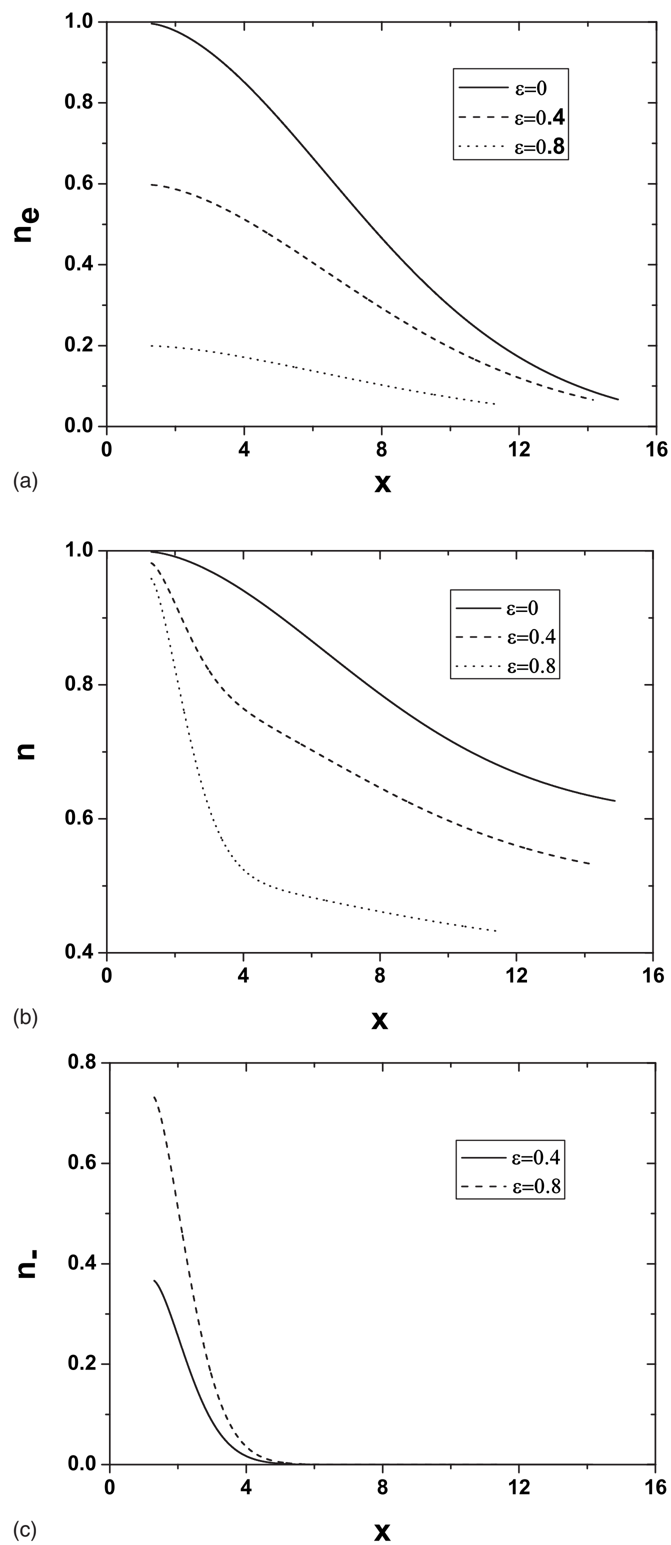

FIG. 1. The spatial profile of the normalized particle densities in the void for the negative-ion concentrations $\epsilon=0,0.4,0.8$. (a) n_e, (b) n_i, and (c) n_-. The ionization mean free path is $x_{0i}=6.67$ and the ion mean free path is $x_n=5$.

void size, as discussed by Tsytovich and Goree.[26] We can also see that as n_- increases, the void size decreases. This is to be expected since as mentioned, larger n_- leads to smaller electric field and smaller ion drift velocity u. Moreover, as Fig. 1(b) shows, the positive-ion density decreases as the content of negative ions increases. A decrease of both the

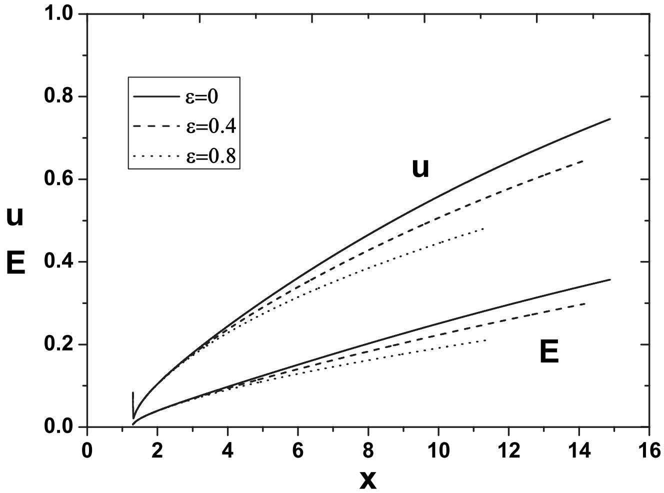

FIG. 2. The normalized electric field $E(x)$ and the ion drift velocity u in the void, for the same parameters as in Fig. 1. The right end (at $x=14.884, 14.159, 11.426$) of each curve indicates the corresponding void boundary.

positive ion velocity and density necessarily leads to a reduction of the outward ion drag force and thus the void size.[34] In Fig. 3(b) the effect of the negative-ion concentration on the dust charge Z_v at the void boundary for different x_n is shown. We see that Z_v increases with x_n, consistent with the results of Ref. 26. More importantly, one can see that as n_- increases, the dust charge $|Z_v|$ first (for small x) increases and then decrease. This is because increasing n_- leads to a decrease of the negative grain currents as well as the positive-ion drift velocity. That is, the positive-ion grain current also decreases. Accordingly, the dust charge $|Z_v|$ first increases. When this occurs, the negative grain current decreases more rapidly than the positive current, so that $|Z_v|$ decreases with a further increase of n_-. Figure 3(c) shows the relation between the ion drift velocity u_v at the void boundary and the negative-ion density for different ion mean free paths x_n. One can see that u_v increases with x_n and decreases with n_-. This can be attributed to the fact that an increase of n_- reduces not only the electric field but also the void size. With weaker and shorter acceleration, the ion velocity at the void edge thus becomes smaller.

IV. CONCLUSION

Using fluid theory, we have studied the effect of negative-ion concentration on the properties of dust voids in collision-dominated plasmas. A model for the quasineutral void in the electronegative plasmas is presented, taking into account ion-neutral collisions. When the ionization rate is sufficiently high, steady-state voids can be sustained by a balance of the outward ion drag force and the inward electrostatic force. Our numerical results show that the charged-particle densities in the void decrease toward the void edge. As the negative-ion concentration increases, the void contracts strongly as the electric field and ion drift velocity within the void and at the boundary decreases. However, because of a competition among the positive and negative grain currents, the dust charge first increases and then decreases with the negative-ion concentration. When applicable, our results are consistent with the existing results[23,26,28,34–36] and should be helpful for the understanding of the coherent structures found in space and laboratory dusty plasmas.

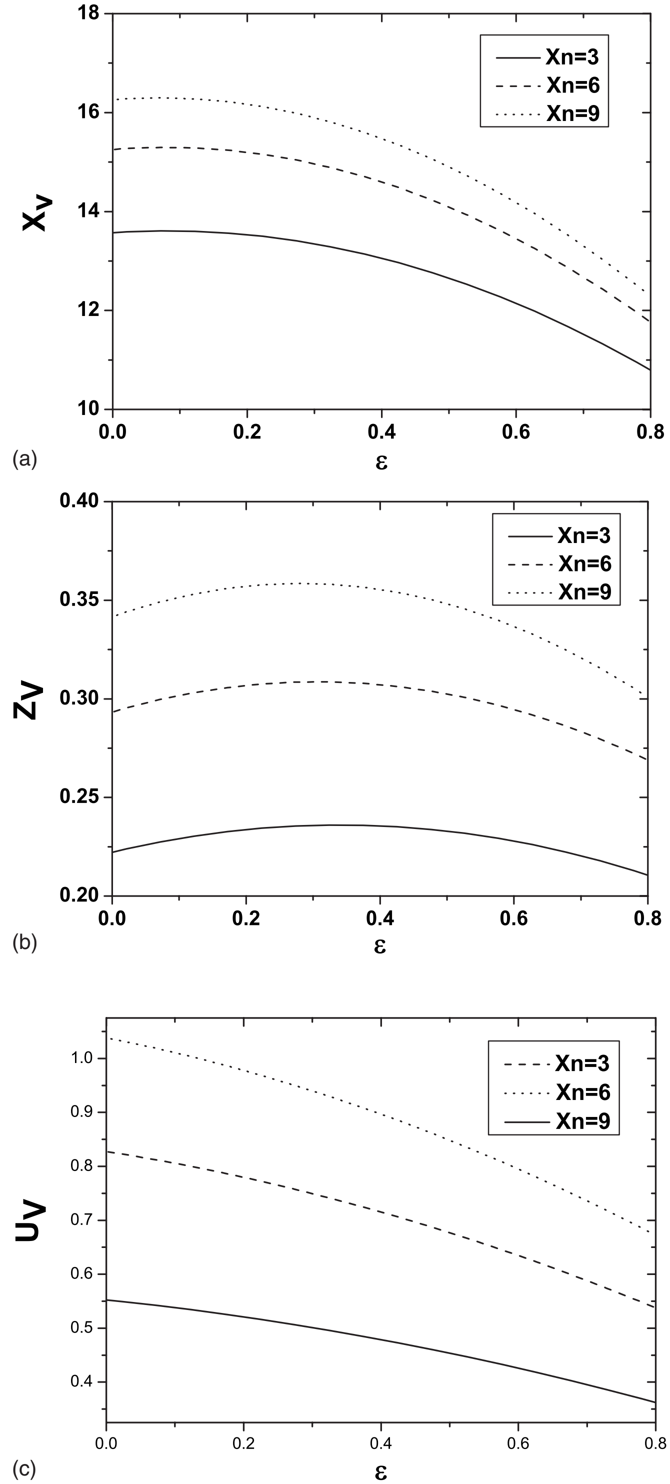

FIG. 3. (a) The void size x_v vs. the negative-ion concentration ϵ for the ion mean free paths $x_n=3,6,9$. The dust charge Z_v and ion drift velocity u_v at the void boundary are shown in (b) and (c), respectively.

Several simplifications have been made in our model of the steady-state quasineutral void. First, the ram and ion thermal pressures that tend to reduce the ion velocity and thus the ion grain current have been neglected and this might affect the drag force on and the charging of the dust grains. Second, we have assumed that negative ions obey the Boltzmann distribution, a condition that is valid only at sufficiently low discharge pressures,[32] so that caution must be taken when applying the results to high-pressure discharges. Third, the temperatures T_e, T_i, and T_- have been assumed to be spatially uniform.[24,26] As pointed out in Refs. 36 and 37 where the distribution of the electron temperature is considered self-consistently, the electron temperature in the void can vary across the dusty plasma and the void. This can affect the local ionization rate and thus the void properties. Thus, caution should be taken when applying our model to plasmas with large temperature difference. Finally, we have precluded direct dust-dust interaction effects and invoked local force balance in determining the void region. These assumptions should be acceptable if the dust density is not so large that the mutual blocking of the grain currents by other dusts can occur.[26] Our results, which are local, should also be applicable to two- or three-dimensional voids if the field distribution (of the discharge) is known.

ACKNOWLEDGMENTS

One of the authors (X.Z.) would like to thank J. Goree for helpful discussions.

This work is supported by the National Natural Science Foundation of China (Grant Nos. 10775134 and 10835003).

[1] J. H. Chu and L. I, Phys. Rev. Lett. **72**, 4009 (1994).
[2] H. Thomas, G. E. Morfill, V. Demmel, J. Goree, B. Feuerbacher, and D. Mohlmann, Phys. Rev. Lett. **73**, 652 (1994).
[3] G. Prabhuram and J. Goree, Phys. Plasmas **3**, 1212 (1996).
[4] J. B. Pieper and J. Goree, Phys. Rev. Lett. **77**, 3137 (1996).
[5] E. Morfill, H. M. Thomas, U. Konopka, H. Rothermel, M. Zuzic, A. Ivlev, and J. Goree, Phys. Rev. Lett. **83**, 1598 (1999).
[6] K. N. Ostrikov, M. Y. Yu, and H. Sugai, J. Appl. Phys. **86**, 2425 (1999).
[7] C. Thompson, N. D'Angelo, and R. L. Merlino, Phys. Plasmas **6**, 1421 (1999).
[8] V. E. Fortov, V. I. Molotkov, A. P. Nefedov, and O. F. Petrov, Phys. Plasmas **6**, 1759 (1999).
[9] D. Samsonov, J. Goree, Z. W. Ma, A. Bhattacharjee, H. Thomas, and G. E. Morfill, Phys. Rev. Lett. **83**, 3649 (1999).
[10] G. Lapenta, Phys. Plasmas **7**, 3822 (2000).
[11] K. N. Ostrikov, M. Y. Yu, and L. Stenflo, Phys. Rev. E **61**, 782 (2000).
[12] S. I. Popel, A. P. Golub, T. V. Losseva, R. Bingham, and S. Benkadda, Phys. Plasmas **8**, 1497 (2001).
[13] K. N. Ostrikov, S. Kumar, and H. Sugai, Phys. Plasmas **8**, 3490 (2001).
[14] A. A. Samarian, S. V. Vladimirov, and B. W. James, Phys. Plasmas **12**, 022103 (2005).
[15] R. Akdim and W. J. Goedheer, Phys. Rev. E **65**, 015401 (2001).
[16] J. G. Xiang, J. C. Lee, and M. A. Nowak, Astrophys. J. **660**, 1309 (2007).
[17] J. C. Weingartner and B. T. Draine, Astrophys. J. **548**, 296 (2001).
[18] K. Ostrikov, Rev. Mod. Phys. **77**, 489 (2005).
[19] C. O. Thompson, N. D'Angelo, and R. L. Merlino, Phys. Plasmas **6**, 1421 (1999).
[20] A. P. Nefedov, G. E. Morfill, V. E. Fortov, H. M. Thomas, H. Rothermel, T. Hagl, A. V. Ivlev, M. Zuzic, B. A. Klumov, A. M. Lipaev, V. I. Molotkov, O. F. Petrov, Y. P. Gidzenko, S. K. Krikalev, W. Shepherd, A. I. Ivanov, M. Roth, H. Binnenbruck, J. A. Goree, and Y. P. Semenov, New J. Phys. **5**, 33 (2003).
[21] M. S. Barnes, J. H. Keller, J. C. Forster, J. A. O'Neill, and D. K. Coultas, Phys. Rev. Lett. **68**, 313 (1992).
[22] D. Bohm, *The Characteristics of Electrical Discharges in Magnetic Fields* (McGraw-Hill, New York, 1949).
[23] D. Samsonov and J. Goree, Phys. Rev. E **59**, 1047 (1999).
[24] J. Goree, G. E. Morfill, V. N. Tsytovich, and S. V. Vladmirov, Phys. Rev. E **59**, 7055 (1999).
[25] G. E. Morfill, H. M. Thomas, U. Konopka, H. Rothermel, M. Zuzic, A. Ivlev, and J. Goree, Phys. Rev. Lett. **83**, 1598 (1999).
[26] V. N. Tsytovich, S. V. Vladimirov, G. E. Morfill, and J. Goree, Phys. Rev. E **63**, 056609 (2001).
[27] K. Avinash, A. Bhattacharjee, and S. Hu, Phys. Rev. Lett. **90**, 075001 (2003).
[28] J. Vyas, G. A. Hebner, and M. J. Kushner, J. Appl. Phys. **92**, 6451 (2002).
[29] I. Denysenko, M. Y. Yu, L. Stenflo, and N. A. Azarenkov, Phys. Plasmas **12**, 042102 (2005).
[30] B. X. Gan, Y. H. Chen, and M. Y. Yu, J. Appl. Phys. **101**, 113310 (2007).
[31] J. Goree (private communication).
[32] M. A. Lieberman and A. J. Lichtenberg, *Principles of Plasma Discharge and Materials Processing* (Wiley, New York, 1994).
[33] N. S. J. Braithwaite and J. E. Allen, J. Phys. D **21**, 1733 (1988).
[34] Y. H. Liu, Z. Y. Chen, F. Huang, M. Y. Yu, L. Wang, and A. Bogaerts, Phys. Plasmas **13**, 052110 (2006).
[35] D. Samsonov, J. Goree, Z. W. Ma, A. Bhattacharjee, H. Thomas, and G. E. Morfill, Phys. Rev. Lett. **83**, 3649 (1999).
[36] I. Denysenko, K. N. Ostrikov, M. Y. Yu, and N. A. Azarenkov, Phys. Rev. E **74**, 036402 (2006).
[37] K. N. Ostrikov, M. Y. Yu, and L. Stenflo, IEEE Trans. Plasma Sci. **27**, 219 (1999).