Vol 18 No 5, May 2009
1674-1056/2009/18(05)/2101-08

Chinese Physics B

Ion pickup by intrinsic low-frequency Alfvén waves with a spectrum*

Lu Quan-Ming(陆全明)[a)b)], Li Xing(李 醒)[c)], and Dong Chuan-Fei(董川飞)[a)]

[a)] *School of Earth and Space Sciences, University of Science and Technology of China, Hefei* 230026, *China*
[b)] *State Key Laboratory of Space Weather, Chinese Academy of Sciences, Beijing* 100080, *China*
[c)] *Institute of Mathematical and Physical Sciences, Aberystwyth University, Aberystwyth, SY23 2BZ, UK*

(Received 22 September 2008; revised manuscript received 19 October 2008)

Ion pickup by a monochromatic low-frequency Alfvén wave, which propagates along the background magnetic field, has recently been investigated in a low beta plasma (Lu and Li 2007 *Phys. Plasmas* **14** 042303). In this paper, the monochromatic Alfvén wave is generalized to a spectrum of Alfvén waves with random phase. It finds that the process of ion pickup can be divided into two stages. First, ions are picked up in the transverse direction, and then phase difference (randomization) between ions due to their different parallel thermal motions leads to heating of the ions. The heating is dominant in the direction perpendicular to the background magnetic field. The temperatures of the ions at the asymptotic stage do not depend on individual waves in the spectrum, but are determined by the total amplitude of the waves. The effect of the initial ion bulk flow in the parallel direction on the heating is also considered in this paper.

Keywords: ion pickup, Alfvén wave, ion heating, temperature anisotropy
PACC: 9420R, 5250G, 5265

1. Introduction

Large amplitude Alfvén waves prevail in the space plasma environment. In such an environment, ion pickup processes occur ubiquitously when neutral particles are ionized by photoionization, impact ionization, or charge change.[1,2] Because there are no electromagnetic forces acting on the neutral particles, a slippage between the ion and neutral populations may occur. Therefore, the newly born ions from neutrals do not have same fluid velocity as the background ions. When the difference between their fluid velocities is sufficiently large, electromagnetic instabilities can be excited and the newly born ions are picked up.[3,4] However, when their velocity difference is not large, the pickup process of newly born ions by intrinsic Alfvén waves may be important.[5]

Recently, we investigated ion pickup processes by a monochromatic, parallel propagating, circularly polarized Alfvén wave.[6,7] It was found that given the electromagnetic field of an Alfvén wave, ions, whose initial average velocity does not match that dictated by the electromagnetic field of the wave, can be heated by such a wave without relying on cyclotron resonance in a low beta plasma. The heating, which is more efficient in the perpendicular direction, is produced by phase difference (randomization) between ions due to their parallel thermal motions. The time scale over which ions are significantly heated can be roughly expressed as π/kv_{th} (where v_{th} is the initial thermal velocity of the ions). In this paper, we extend the monochromatic Alfvén wave to waves with a spectrum, and the Alfvén waves have random phases. Analytical theory as well as test particle calculations are used in this paper to investigate the ion pickup processes by the intrinsic Alfvén waves. It is found that the total kinetic energy gained by the pickup ions can be separated into two parts. The first part corresponds to the linear fluid velocity perturbations induced by the Alfvén waves. The other part can be transferred into thermal energy (random motions) due to phase difference between ions as in the case of the monochromatic Alfvén wave, eventually leading to the heating of the ions. We also consider the effects of the initial ion bulk flow in the parallel direction on the ion pickup processes.

The paper is organized as follows. In Section 2, the results of the analytical theory are presented. The results of test particle calculations are described in Section 3. Finally, the discussions and conclusions are

*Project supported by the National Natural Science Foundation of China (Grants Nos 40725013 and 40674093) and Chinese Academy of Sciences (Grant No KJCX2-YW-N28 9140C08060507ZCZJ19).

given in Section 4.

2. Analytical theory

The Alfvén waves considered in this paper are assumed to propagate along the background magnetic field $\boldsymbol{B}_0 = B_0\boldsymbol{i}_z$, and they are circularly polarized with a left-handed sense. The Alfvén waves have a spectrum, and the dispersion relation can be described as $\omega = kv_{\rm A}$ ($v_{\rm A}$ is the Alfvén speed, ω and k are the wave angular frequency of wave and wave number respectively). The total magnetic field $\delta\boldsymbol{B}_w$ and electric field $\delta\boldsymbol{E}_w$ of wave can be expressed as

$$\delta\boldsymbol{B}_w = \sum_k B_k(\cos\phi_k\boldsymbol{i}_x - \sin\phi_k\boldsymbol{i}_y), \tag{1}$$

$$\delta\boldsymbol{E}_w = -\frac{v_{\rm A}}{c}\,\boldsymbol{b}\times\delta\boldsymbol{B}_w, \boldsymbol{b} = \frac{\boldsymbol{B}_0}{B_0}, \tag{2}$$

where $\boldsymbol{i}_x$ and $\boldsymbol{i}_y$ are unit directional vectors, $\phi_k = k(v_{\rm A}t - z) + \varphi_k$ and φ_k is the random phase for mode k. The motion of a particle in the Alfvén waves can be described by

$$m_{\rm j}\frac{{\rm d}\boldsymbol{v}}{{\rm d}t} = e_{\rm j}\left[\delta\boldsymbol{E}_w + \frac{\boldsymbol{v}}{c}\times(B_0\boldsymbol{i}_z + \delta\boldsymbol{B}_w)\right], \tag{3}$$

$$\frac{{\rm d}z}{{\rm d}t} = v_z, \tag{4}$$

where $e_{\rm j}$ is the charge of the particle. For convenience, we introduce the following notations: $u_\perp = v_x + {\rm i}v_y$, and $v_\parallel = v_z$. Then,

$$\frac{{\rm d}u_\perp}{{\rm d}t} + {\rm i}\varOmega_0 u_\perp = {\rm i}(v_\parallel - v_{\rm A})\sum_k \varOmega_k{\rm e}^{-{\rm i}\phi_k}, \tag{5}$$

$$\frac{{\rm d}v_\parallel}{{\rm d}t} = -{\rm Im}\left(u_\perp\sum_k \varOmega_k{\rm e}^{-{\rm i}\phi_k}\right), \quad \frac{{\rm d}z}{{\rm d}t} = v_\parallel, \tag{6}$$

where $\varOmega_0 = e_{\rm j}B_0/m_{\rm j}c$, $\varOmega_k = e_{\rm j}B_k/m_{\rm j}c$, and the subscript j refers to ion species. Im() denotes the imaginary part of its argument. As an approximation, we assume that $v_\parallel$ is a constant $v_\parallel \approx v_\parallel(0)$, where $v_\parallel(0)$ is the particle's initial parallel velocity. The approximation is valid when $\sum_k B_k/B_0$ is sufficiently small and the frequencies of the Alfvén waves are sufficiently low, so $|\varOmega_0| \gg |k[v_\parallel - v_{\rm A}]|$ for each mode. With the initial condition $u_\perp = u_\perp(0)$ and $z = z(0)$, the solution of Eq.(5) can be written as[8,9]

$$u_\perp = u_\perp(0){\rm e}^{-{\rm i}\varOmega_0 t} - [v_{\rm A} - v_\parallel(0)]\sum_k\frac{\varOmega_k}{\tilde{\varOmega}_k}{\rm e}^{-{\rm i}k[v_{\rm A}-v_\parallel(0)]t+{\rm i}kz(0)-{\rm i}\varphi_k} + [v_{\rm A} - v_\parallel(0)]\sum_k\frac{\varOmega_k}{\tilde{\varOmega}_k}{\rm e}^{{\rm i}[kz(0)-\varphi_k]}{\rm e}^{-{\rm i}\varOmega_0 t}, \tag{7}$$

where $\tilde{\varOmega}_k = \varOmega_0 - k[v_{\rm A} - v_\parallel(0)] \approx \varOmega_0$ and $z = z(0) + v_\parallel(0)t$. Then

$$u_\perp = u_\perp(0){\rm e}^{-{\rm i}\varOmega_0 t} - [v_{\rm A} - v_\parallel(0)]\sum_k\frac{B_k}{B_0}{\rm e}^{-{\rm i}k(v_{\rm A}t-z)-{\rm i}\varphi_k} + [v_{\rm A} - v_\parallel(0)]\sum_k\frac{B_k}{B_0}{\rm e}^{{\rm i}[kz(0)-\varphi_k]}{\rm e}^{-{\rm i}\varOmega_0 t}. \tag{8}$$

The above equation describes the transverse motion of the particle, and it consists of three parts: the first is the gyromotion of the particle in the background magnetic field; the second is the transverse motion due to the electric field of the Alfvén waves; and the third is the modification of the above gyromotion due to the existence of the Alfvén waves.

The above analysis is for a single particle. Now let us consider an ensemble of particles at a fixed z position in a low beta plasma. Assume $v_\parallel(0) = v_{\rm b} + v_\parallel{}'(0)$, where $v_{\rm b}$ is the initial bulk flow speed of the particles in the parallel direction, and $v_\parallel{}'(0)$ corresponds to the initial thermal velocity in the parallel direction. Initially, the average transverse velocity of these particles is set to be zero everywhere. In a low beta plasma, $v_\parallel{}'(0) \ll v_{\rm A}$, then $v_{\rm A} - v_\parallel(0) \approx v_{\rm A} - v_{\rm b}$. Therefore, Eq.(8) can be rewritten as

$$\begin{aligned} u_\perp &= u_\perp(0){\rm e}^{-{\rm i}\varOmega_0 t} - (v_{\rm A} - v_{\rm b})\sum_k\frac{B_k}{B_0}{\rm e}^{-{\rm i}k(v_{\rm A}t-z)-{\rm i}\varphi_k} \\ &\quad + (v_{\rm A} - v_{\rm b})\sum_k\frac{B_k}{B_0}{\rm e}^{{\rm i}k[z-v_{\rm b}t-v_\parallel{}'(0)t]-{\rm i}\varphi_k}{\rm e}^{-{\rm i}\varOmega_0 t}. \end{aligned} \tag{9}$$

In Eq.(9), the first term is independent of position z. The corresponding thermal velocity of the particles will be kept as their initial value, while the average transverse velocity is zero. From the second term we can know that all particles at position z have the same velocity, and there is no thermal dispersion. In the third term, particles with initial position

$z(0)=z-v_{\parallel}(0)t$ will arrive at position z after a finite time t. In a homogeneous plasma, whenever a particle arrives at position z, another particle with the same parallel velocity will leave position z simultaneously. Therefore, the parallel velocity distribution of the particles at position z will not change. If we also assume that initially particles have a Maxwellian velocity distribution function in the parallel direction, the corresponding average transverse velocity at position z can be approximated as $1/(\sqrt{\pi}\, v_{\rm th})\,(v_{\rm A}-v_{\rm b})\int_{-\infty}^{\infty}\sum_k B_k/B_0\,{\rm e}^{{\rm i}\,k\,[z-v_{\rm b}t+v_{\parallel}{}'(0)t]-{\rm i}\,\varphi_k}\,{\rm e}^{-{\rm i}\,\Omega_0\,t}\,{\rm e}^{-[v_{\parallel}{}'(0)/v_{\rm th}]^2}{\rm d}v_{\parallel}{}'(0)$.

Summing up the three terms, we can find the overall average transverse velocity at position z as

$$\begin{aligned} &U_{\perp}\\ &=-(v_{\rm A}-v_{\rm b})\sum_k \frac{B_k}{B_0}{\rm e}^{-{\rm i}k(v_{\rm A}t-z)-{\rm i}\varphi_k}+\frac{1}{\sqrt{\pi}v_{\rm th}}(v_{\rm A}-v_{\rm b})\int_{-\infty}^{\infty}\sum_k\frac{B_k}{B_0}{\rm e}^{{\rm i}k[z-v_{\rm b}t-v_{\parallel}{}'(0)t]-{\rm i}\varphi_k}{\rm e}^{-{\rm i}\Omega_0 t}{\rm e}^{-\left[\frac{v_{\parallel}{}'(0)}{v_{\rm th}}\right]^2}{\rm d}v_{\parallel}{}'(0)\\ &=-(v_{\rm A}-v_{\rm b})\sum_k \frac{B_k}{B_0}{\rm e}^{-{\rm i}k(v_{\rm A}t-z)-{\rm i}\varphi_k}+(v_{\rm A}-v_{\rm b})\sum_k A_k\frac{B_k}{B_0}{\rm e}^{{\rm i}k(z-v_{\rm b}t)-{\rm i}\varphi_k}{\rm e}^{-{\rm i}\Omega_0 t}, \end{aligned} \tag{10}$$

where $A_k=(1/\sqrt{\pi})\int_{-\infty}^{\infty}\cos(kv_{\rm th}tx){\rm e}^{-x^2}{\rm d}x={\rm e}^{-k^2v_{\rm th}^2t^2/4}$, and $v_{\rm th}=(2k_{\rm B}T_{j0}/m_{\rm j})^{1/2}$ (here T_{j0} is the initial temperature of ion species j) is the particle's initial thermal speed. Therefore, the perpendicular temperature is

$$\begin{aligned} &T_{\perp \rm j}\\ &=\frac{m_{\rm j}}{2k_{\rm B}v_{\rm th}\sqrt{\pi}}\int|u_{\perp}-U_{\perp}|^2{\rm e}^{\left[\frac{v_{\parallel}{}'(0)}{v_{\rm th}}\right]^2}{\rm d}v_{\parallel}{}'(0)\\ &=T_{\rm j0}+\frac{m_{\rm j}(v_{\rm A}-v_{\rm b})^2}{2k_{\rm B}}\sum_k\frac{B_k^2}{B_0^2}(1-A_k^2)\\ &=T_{\rm j0}\left[1+\left(1-\frac{v_{\rm b}}{v_{\rm A}}\right)^2\frac{m_{\rm j}}{m_{\rm p}\beta_{\rm j}}\sum_k\frac{B_k^2}{B_0^2}(1-A_k^2)\right], \end{aligned} \tag{11}$$

where $\beta_{\rm j}=8\pi n_{\rm e}k_{\rm B}T_{\rm j0}/B_0^2$ is the plasma beta of species j, and $n_{\rm e}$ is the electron number density. In deriving Eq.(11), we have adopted the following random phase approximation:[8]

$$\sum_k\sum_{k'}{\rm e}^{-{\rm i}\varphi_k}{\rm e}^{{\rm i}\varphi_{k'}}=\sum_k\sum_{k'}\delta(k-k'). \tag{12}$$

The increase of the perpendicular temperature is due to the initial random velocities of particles in the parallel direction. According to the third term on the right-hand side of Eq.(9), particles at position z will have different velocities after time t due to the phase difference between particles. As a result, a velocity dispersion will be produced and ions will be heated in the perpendicular direction. The heating process saturates when the phase difference among particles with characteristic speed $v_{\rm th}$ reaches π.

Now let us consider the particles' parallel motion. After substituting Eq.(9) into Eq.(6), we can obtain

$$\begin{aligned} &v_{\parallel}\\ &=v_{\parallel}(0)-v_{\perp}(0)\sum_k\frac{\Omega_k}{\tilde{\Omega}_k}[\cos(\Phi_k(0)-\tilde{\Omega}_kt)-\cos\Phi_k(0)]\\ &\quad-[v_{\rm A}-v_{\parallel}(0)]\sum_k\frac{\Omega_k^2}{\tilde{\Omega}_k^2}[\cos(\tilde{\Omega}_kt)-1] \end{aligned} \tag{13}$$

where $\Phi_k(0)=\alpha(0)+[\phi_k-kz(0)]$, and $\alpha(0)=\arctan[v_y(0)/v_x(0)]$ is the initial gyrophase angle. In deriving the above equation, the random phase approximation given in Eq.(12) is also necessary. Considering $|\Omega_0|\gg|k[v_{\parallel}(0)-v_{\rm A}]|$, $v_{\parallel}(0)=v_{\rm b}+v'_{\parallel}(0)\approx v_{\rm b}$, $|v_{\perp}(0)|\ll v_{\rm A}$ in a low beta plasma, we obtain

$$\begin{aligned} v_{\parallel}&=v_{\parallel}(0)+(v_{\rm A}-v_{\rm b})\sum_k\frac{B_k^2}{B_0^2}\\ &\quad\times\left\{1-\cos[\Omega_0t-kv_{\rm A}t+kv_{\parallel}(0)t]\right\}. \end{aligned} \tag{14}$$

Therefore, the average parallel velocity is

$$\begin{aligned} U_{\parallel}&=v_{\rm b}+\frac{(v_{\rm A}-v_{\rm b})}{v_{\rm th}\sqrt{\pi}}\int_{-\infty}^{\infty}\sum_k\frac{B_k^2}{B_0^2}\{1-\cos[\Omega_0t-kv_{\rm A}t+kv_{\rm b}t+kv_{\parallel}{}'(0)t]\}{\rm e}^{-\left[\frac{v_{\parallel}{}'(0)}{v_{\rm th}}\right]^2}{\rm d}v_{\parallel}{}'(0)\\ &=v_{\rm b}+(v_{\rm A}-v_{\rm b})\sum_k\frac{B_k^2}{B_0^2}[1-A_k\cos(\Omega_0t-kv_{\rm A}t+kv_{\rm b}t)], \end{aligned} \tag{15}$$

when $t \to \infty$, $A_k \to 0$. Therefore, the asymptotic values of the average parallel and transverse velocity, and the perpendicular temperature, are respectively

$$U_\parallel = v_{\rm b} + (v_{\rm A} - v_{\rm b}) \sum_k \frac{B_k^2}{B_0^2}, \tag{16}$$

$$U_\perp = -(v_{\rm A} - v_{\rm b}) \sum_k \frac{B_k}{B_0} {\rm e}^{-{\rm i}k(v_{\rm A}t - z) - {\rm i}\varphi_k}, \tag{17}$$

$$T_{\perp{\rm j}} = T_{\rm j0} \left[1 + \left(1 - \frac{v_{\rm b}}{v_{\rm A}} \right)^2 \frac{m_{\rm j}}{m_{\rm p}\beta_{\rm j}} \sum_k \frac{B_k^2}{B_0^2} \right]. \tag{18}$$

It is well known that in the frame of an Alfvén wave's phase speed the particle motion can be described by

$$(v_\parallel - v_{\rm A})^2 + v_\perp^2 = (v_\parallel(0) - v_{\rm A})^2 + v_\perp^2(0). \tag{19}$$

According to Eq.(18), we can calculate the asymptotic value of the parallel temperature,

$$T_{\parallel{\rm j}} = T_{\rm j0} \left[1 + \left(1 - \frac{v_{\rm b}}{v_{\rm A}} \right)^2 \frac{m_{\rm j}}{m_{\rm p}\beta_{\rm j}} \left(\sum_k \frac{B_k^2}{B_0^2} \right)^2 \right]. \tag{20}$$

Therefore, the asymptotic value of the temperature anisotropy is

$$\frac{T_{\perp{\rm j}}}{T_{\parallel{\rm j}}} = \frac{1 + \left(1 - \frac{v_{\rm b}}{v_{\rm A}} \right)^2 \frac{m_{\rm j}}{m_{\rm p}\beta_{\rm j}} \sum_k \frac{B_k^2}{B_0^2}}{1 + \left(1 - \frac{v_{\rm b}}{v_{\rm A}} \right)^2 \frac{m_{\rm j}}{m_{\rm p}\beta_{\rm j}} \left(\sum_k \frac{B_k^2}{B_0^2} \right)^2}. \tag{21}$$

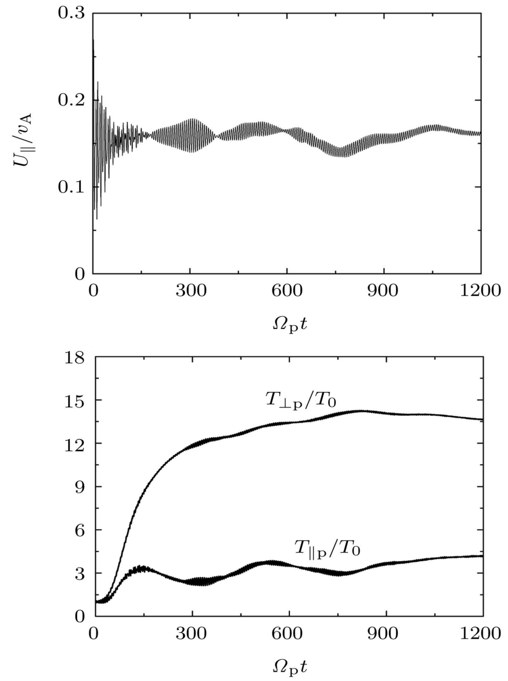
3. Calculations about the test particle

In the above analytical theory, one basic approximation in the treatment of the non-resonant interaction between ions and Alfvén waves is that the amplitude of the Alfvén waves is sufficiently small. Therefore, the parallel velocity of the particle can be considered to be kept at its initial value. In this section, we conduct calculations with test particles to prove the validity of the theory for the case of finite amplitudes of the waves. In the calculations, the motions of the particles in the Alfvén waves are governed by Eqs.(3) and (4). The equations are solved with the Boris algorithm. The time step is 0.03 Ω_0^{-1}. Initially, particles are evenly distributed in a region of length 2048 $V_{\rm A}\Omega_0^{-1}$, and the total number of particles is 307200.

The intrinsic Alfvén waves are described by Eqs.(1) and (2). They propagate along the ambient magnetic field in the z-direction, and their total amplitude is $\sum_k \delta B_k^2/B_0^2 = 0.16$. The frequencies of the waves extend from $\omega_1 = 0.05\ \Omega_0$ to $\omega_N = 0.1\ \Omega_0$, and N is the number of wave modes used in our calculations. We choose $N = 100$, and $\omega_i = \omega_1 + (i-1)\Delta\omega (i = 1, \ldots, N)$, where $\Delta\omega = (\omega_N - \omega_1)/(N-1)$. The amplitude of individual wave modes satisfies the relation $B_i/B_1 = (\omega_i/\omega_1)^{-q/2}$, and q is chosen as 1.667. This means that the power spectrum of the Alfvén waves has an index of 1.667, a generally accepted value for the power of magnetic fluctuations found in *in situ* measurements in the solar wind.[10,,11] We choose $\beta_{\rm p} = \beta_{\rm O} = 0.01$, and initially O^{5+} ions have the same temperature T_0 as protons. In this and the following section, we calculate the average velocity, and the parallel and perpendicular temperatures using the following procedure: we firstly estimate $U_\parallel = \langle v_z \rangle$, $T_\parallel = (m_i/k_{\rm B}) \left\langle (v_z - \langle v_z \rangle)^2 \right\rangle$, $T_{x,y} = (m_i/k_{\rm B}) \left\langle (v_{x,y} - \langle v_{x,y} \rangle)^2 \right\rangle$, and $T_\perp = 0.5(T_x + T_y)$ in every grid cell (where the bracket $\langle . \rangle$ denotes an average over a grid cell), and then these quantities are averaged over all grid cells. In this way, we can eliminate the effects of the average velocity at each location on the thermal temperature.

Figure 1 shows the time evolution of the average parallel velocity $U_\parallel/v_{\rm A}$, the parallel temperature $T_{\parallel{\rm p}}/T_0$, the perpendicular temperature $T_{\perp{\rm p}}/T_0$, and the temperature anisotropy $T_{\perp{\rm p}}/T_{\parallel{\rm p}}$ for protons. Consistent with the analytical theory, protons can be significantly heated, and the heating is more efficient in the perpendicular direction. Therefore protons can obtain a large temperature anisotropy. Since a particle's kinetic energy is conserved in the frame of the Alfvén phase speed, particles can obtain a bulk velocity in the parallel direction after they are heated in the perpendicular direction. The characteristic time scale over which protons are significantly heated is about $\pi/(kv_{\rm th}) = (\pi/\omega)\sqrt{m_{\rm j}/(m_{\rm p}\beta_{\rm j})}$. For the highest frequency in the wave spectrum $\omega_N = 0.1\Omega_{\rm p}$, the characteristic time scale is about 314 $\Omega_{\rm p}^{-1}$, while the the characteristic time scale is about 628 $\Omega_{\rm p}^{-1}$ for the lowest frequency $\omega_1 = 0.05\ \Omega_{\rm p}$. Correspondingly, from the figure, we can find that protons are significantly heated at about 314 $\Omega_{\rm p}^{-1}$, and they are continuously heated until at about 628 $\Omega_{\rm p}^{-1}$. At the asymptotic stage, $U_\parallel/v_{\rm A}$, $T_{\parallel{\rm p}}/T_0$, $T_{\perp{\rm p}}/T_0$ and $T_{\perp{\rm p}}/T_{\parallel{\rm p}}$ are about 0.16, 4.0, 13.5 and 3.2, respectively. Based on the analytic theory in the above section, we can find that the corresponding values are 0.16, 3.56, 17.0, and 4.7, respectively.

The heating process of protons can also be shown in Fig.2, which displays the scattered plots of protons between 500 $v_{\mathrm{A}}\Omega_{\mathrm{p}}^{-1}$ and 756 $v_{\mathrm{A}}\Omega_{\mathrm{p}}^{-1}$ at $\Omega_{\mathrm{p}}t = 0$, 60, and 900. Figures 2(a) and 2(b) depict the velocity component in the parallel and y directions, respectively. Initially, the velocity distribution of protons is Maxwellian with the thermal speed $v_{\mathrm{th}} = 0.1v_{\mathrm{A}}$. At $\Omega_{\mathrm{p}}t = 60$, protons are trapped by the electric field of the Alfvén wave and the average velocities are obtained in both the parallel and perpendicular directions. However, there is no obvious heating at this time. At $\Omega_{\mathrm{p}}t = 900$, significant heating can be found, and it is more efficient in the perpendicular direction.



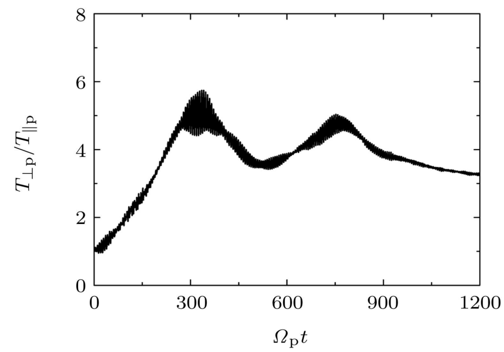


Fig.1. The time evolution of the average parallel velocity $U_{\parallel}/v_{\mathrm{A}}$, the parallel temperature $T_{\parallel\mathrm{p}}/T_0$, the perpendicular temperature $T_{\perp\mathrm{p}}/T_0$, and the temperature anisotropy $T_{\perp\mathrm{p}}/T_{\parallel\mathrm{p}}$ for protons.

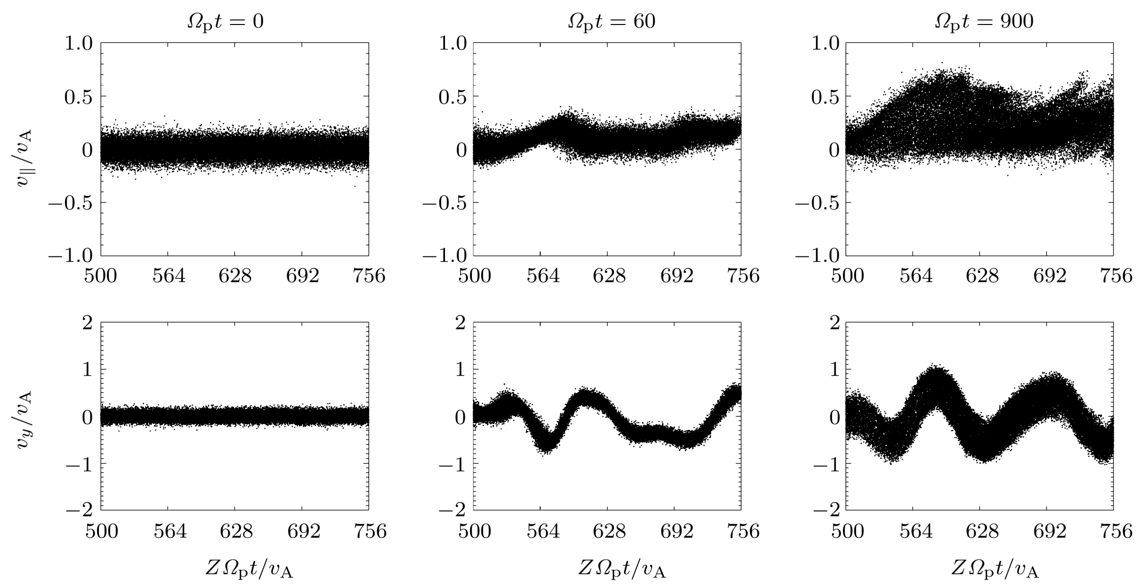


Fig.2. The scattered plots of protons between 500 $v_{\mathrm{A}}\Omega_{\mathrm{p}}^{-1}$ and 756 $v_{\mathrm{A}}\Omega_{\mathrm{p}}^{-1}$ at $\Omega_{\mathrm{p}}t = 0$, 60, and 900.

Figure 3 shows the evolution of the velocity distribution of protons in a small region (the size of the region is significantly smaller than the shortest wavelength in the wave spectrum) between 697 $v_{\mathrm{A}}\Omega_{\mathrm{p}}^{-1}$ and 703 $v_{\mathrm{A}}\Omega_{\mathrm{p}}^{-1}$. Initially, protons satisfy the Maxwellian distribution, and they are dramatically scattered in the transverse direction in the phase space. Ring velocity distributions are nicely shown in the figure. The ring velocity distributions are unstable. They may eventually be thermalized by relevant microscopic instabilities.[12,13] The formation of a ring velocity distribution was discovered in Ref.[7] and can be easily understood. From Eqs.(9) and (10), we can find that in low beta plasma $|u_\perp - U_\perp|$ can be approximated as $(v_\mathrm{A} - v_\mathrm{b})\sqrt{\sum_k {B_k}^2/{B_0}^2}$ at the asymptotic stage. It means that the transverse velocity in the wave frame satisfies ring distribution, and the radius is $(v_\mathrm{A} - v_\mathrm{b})\sqrt{\sum_k {B_k}^2/{B_0}^2}$. The results are similar to that in the case of a monochromatic Alfvén wave.[6]

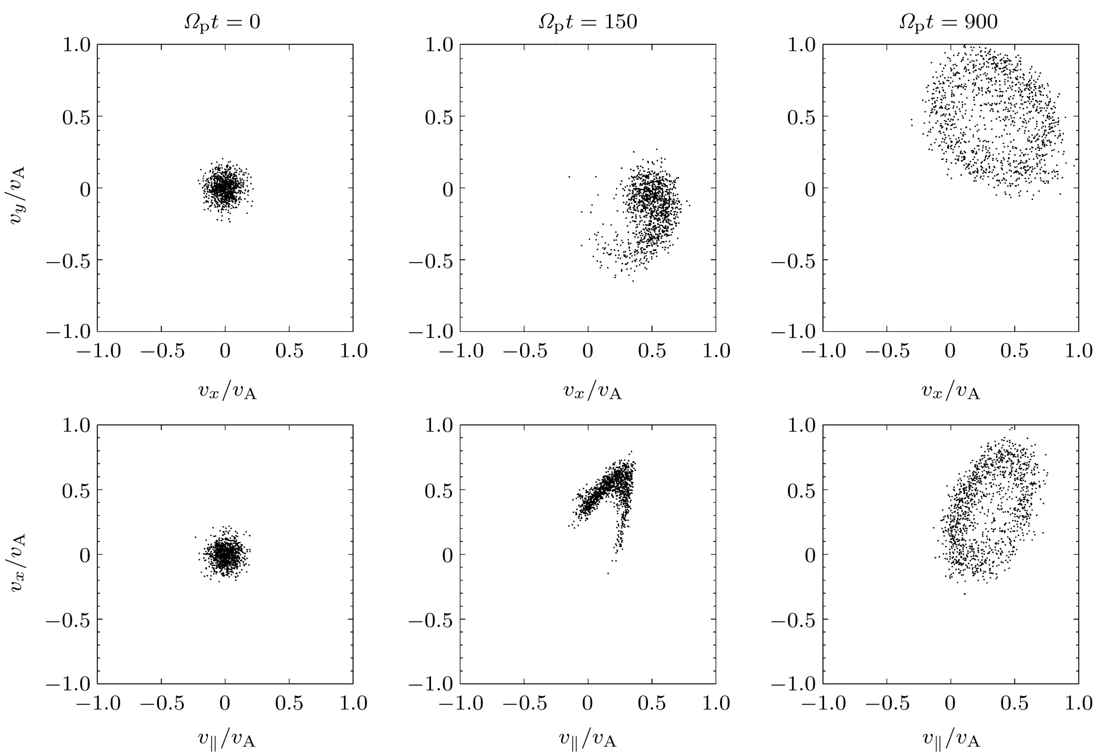
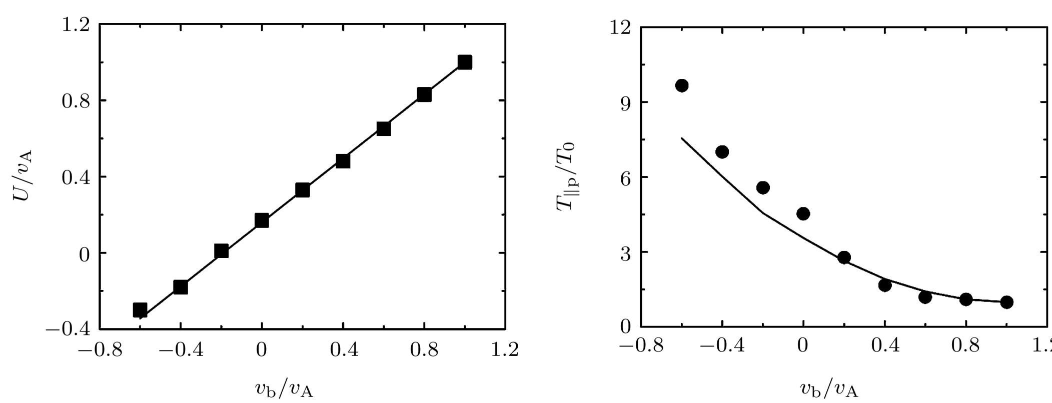


Fig.3. The evolution of velocity distribution of protons in a small region between 697 $v_\mathrm{A}\Omega_\mathrm{p}^{-1}$ and 703 $v_\mathrm{A}\Omega_\mathrm{p}^{-1}$.

We also calculate the heating of protons for different initial parallel bulk velocities. Figure 4 displays the average parallel velocity $U_\parallel/v_\mathrm{A}$, the parallel temperature $T_{\parallel\mathrm{p}}/T_0$, and the perpendicular temperature $T_{\perp\mathrm{p}}/T_0$ of protons at the asymptotic stage as a function of v_b. With the increase of the initial parallel bulk velocity, the heating becomes less efficient in both parallel and perpendicular directions.



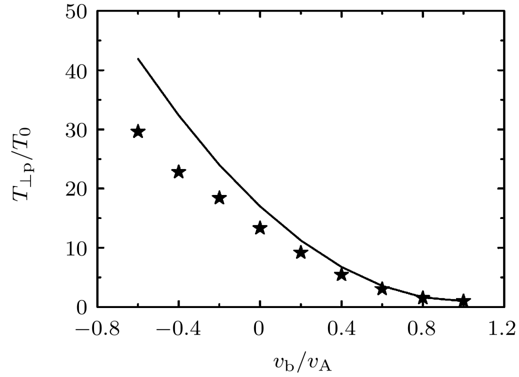


Fig.4. The average parallel velocity $U_\parallel/v_A$, the parallel temperature $T_{\parallel p}/T_0$, and the perpendicular temperature $T_{\perp p}/T_0$ of protons at the asymptotic stage for different initial parallel bulk velocities v_b. In the figure, the solid lines are based on the analytical theory. The square, circle and star are based on the calculations with test particles.

From the analytical theory in Section 2, the heating is more efficient for heavy ions. Figure 5 shows the time evolution of the average parallel velocity $U_\parallel/v_A$, the parallel temperature $T_{\parallel O}/T_0$, the perpendicular temperature $T_{\perp O}/T_0$, and the temperature anisotropy $T_{\perp O}/T_{\parallel O}$ for O^{5+}. At about $\Omega_p t = 2512$, the asymptotic stage is attained. The average parallel velocity $U_\parallel/v_A$, the parallel temperature $T_{\parallel O}/T_0$, the perpendicular temperature $T_{\perp O}/T_0$, and the temperature anisotropy $T_{\perp O}/T_{\parallel O}$ at the asymptotic stage are about 0.17, 30, 136, and 4.5, respectively. According to the analytical theory, the corresponding values are about 0.16, 42, 257, and 6.1, respectively.

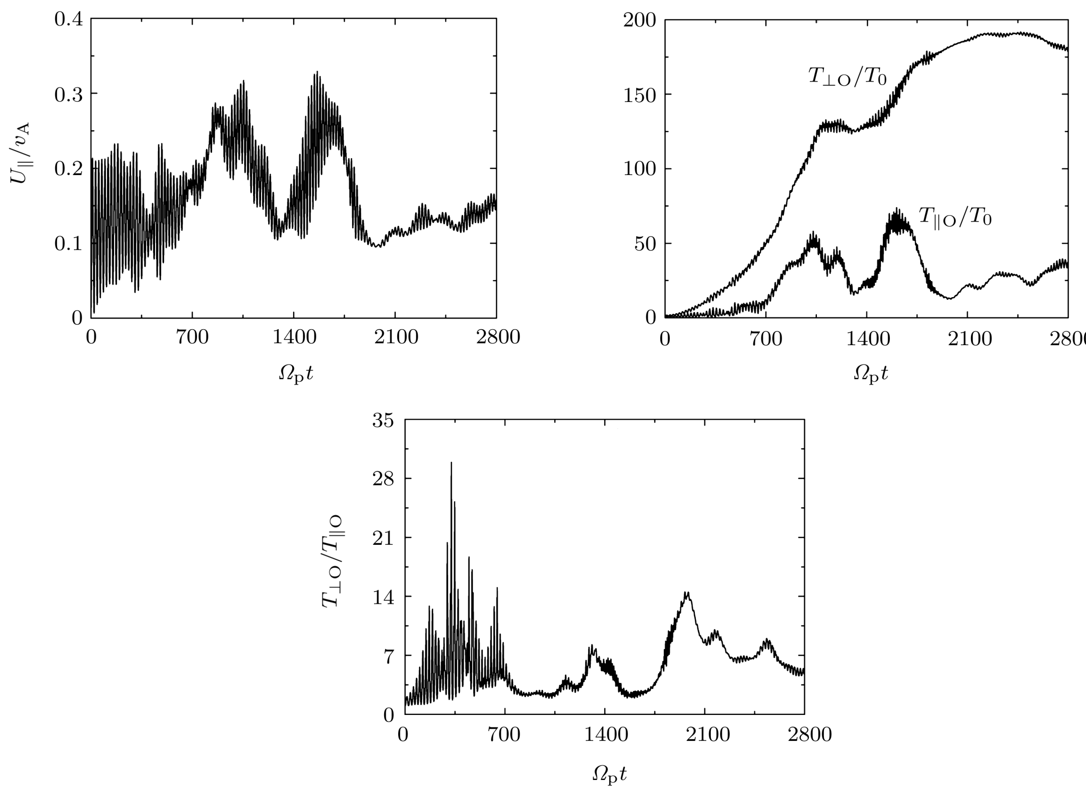


Fig.5. The time evolution of the average parallel velocity $U_\parallel/v_A$, the parallel temperature $T_{\parallel O}/T_0$, the perpendicular temperature $T_{\perp O}/T_0$, and the temperature anisotropy $T_{\perp O}/T_{\parallel O}$ for O^{5+}.

We have also changed the index of the wave spectrum while keeping the amplitude $\sum_k \delta B_k^2/B_0^2 = 0.16$ unchanged to investigate their effects on the heating for both protons and O^{5+}. Consistent with the analytical theory, no obvious changes can be found from the calculations (not shown).

4. Discussions and conclusions

Recently, Lu and Li[6] investigated ion pickup processes by a monochromatic low-frequency Alfvén wave propagating along the background magnetic field. They found that ions can be heated in a low beta plasma, and the heating is more efficient in the perpendicular direction, therefore producing a large temperature anisotropy. The heating is generated by the phase difference between ions due to the existence of the Alfvén wave. In this paper, the study is generalized to a spectrum of Alfvén waves. Ions can also be heated by a spectrum of Alfvén waves with the same mechanism. Both the parallel and perpendicular temperatures at the asymptotic stage are independent of the amplitude of individual waves in the spectrum or the power index of the spectrum if the total amplitude $\sum_k \delta B_k^2/B_0^2$ is kept as a constant, and detailed structure of the wave spectrum only influences the time evolution of the temperatures. The heating of ions is more efficient with a larger ion mass, while it is less efficient when the initial parallel bulk velocity of the ion is comparable with the Alfvén speed.

The processes of ion pickup can be divided into two stages. At first the newly born particles are picked up by the Alfvén waves and gain a kinetic energy which is largely due to the bulk velocity of the particles. This kinetic energy can be separated into two parts. The first part of the energy corresponds to the linear fluid velocity perturbations induced by the Alfvén waves. The remaining part will be transferred into random motions of ions at a later stage through phase mixing, which leads to the heating of ions. According to the results in this paper, the distribution functions of particles in the first few proton gyration periods do not change (but they shift due to $U_\perp$) and particles are not heated at all.

The mechanism, which heats ions via ion pickup processes, may have relevance in the study of the heating of solar corona. The results obtained in this paper are consistent with observations in the solar corona: ions with larger mass tend to have higher temperatures.[14,15] Of course, our calculations with test particles ignore the influences of particles on the wave, at the same time the ring distribution of ions obtained in our calculations about the test particle may be unstable to ion cyclotron waves. A self-consistent particle simulation is necessary to overcome these limitations. This will be investigated in a future publication.

References

[1] Gloeckler G, Hovestadt D, Ipavich F M, Scholer M, Klecker B and Galvin A B 1986 *Geophys. Res. Lett.* **13** 251

[2] Terasawa T, Mukai T, Kitayama M, Miyake W and Hirao K 1986 *Geophys. Res. Lett.* **13** 837

[3] Tsurutani B T and Smith E J 1986 *Geophys. Res. Lett.* **13** 263

[4] Gary S P, Christian C D, Winske D and Omidi N 1988 *J. Geophys. Res.* **93** 9584

[5] Wu C S, Gaffey J D and Winske D 1986 *Geophys. Res. Lett.* **13** 865

[6] Lu Q M and Li X 2007 *Phys. Plasmas* **14** 042303

[7] Li X, Lu Q M and Li B 2007 *Astrophys. J.* **661** L105

[8] Wu C S, Yoon P H and Chao J K 1997 *Phys. Plasmas* **4** 856

[9] Lu Q M, Wu C S and Wang S 2006 *Astrophys. J.* **638** 1169

[10] Villante U 1980 *J. Geophys. Res.* **85** 6869

[11] Bavassano B and Smith E J 1986 *J. Geophys. Res.* **91** 1706

[12] Lu Q M and Wang S 2005 *Geophys. Res. Lett.* **32** L03111

[13] Lu Q M and Wang S 2006 *J. Geophys. Res.* **111** A05204

[14] Kohl J L, Noci G, Antonucci E,Tondello G,Huber M C E, Cranmer S R, Strachan L, Panasyuk A V,Gardner L D, Romoli M, Fineschi S, Dobrzycka D, Raymond J C, Nicolosi P, Siegmund O H W, Spadaro D, Benna C, Ciaravella A, Giordano S, Habbal S R, Karovska M, Li X, Martin R, Michels J G, Modigliani A, Naletto G, O'Neal R H, Pernechele C, Poletto G, Smith P L and Suleiman R M 1998 *Astrophys. J.* **501** L127

[15] Li X, Habbal S R, Kohl J L and Noci G 1998 *Astrophys. J.* **501** L133