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Effects of prediction feedback in multi-route intelligent traffic systems[☆]

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ABSTRACT

We first study the influence of an efficient feedback strategy named the prediction feedback strategy (PFS) based on a multi-route scenario in which dynamic information can be generated and displayed on the board to guide road users to make a choice. In this scenario, our model incorporates the effects of adaptability into the cellular automaton models of traffic flow. Simulation results adopting this optimal information feedback strategy have demonstrated high efficiency in controlling spatial distribution of traffic patterns compared with the other three information feedback strategies, i.e., vehicle number and flux. At the end of this paper, we also discuss in what situation PFS will become invalid in multi-route systems.

1. Introduction

Vehicular traffic flow and related problems have triggered great interest in a community of physicists in recent years because of the various complex behaviors [1–3]. A lot of theories have been proposed, such as the car-following theory [4], kinetic theory [5–11] and particle-hopping theory [12,13]. These theories have the advantages of alleviating the traffic congestion and enhancing the capacity of existing infrastructure. Although the dynamics of traffic flow with real-time traffic information have been extensively investigated [14–19], finding a more efficient feedback strategy is an all-important task. Recently, some real-time feedback strategies have been proposed, such as the Travel Time Feedback Strategy (TTFS) [14,20], Mean Velocity Feedback Strategy (MVFS) [14,21], Congestion Coefficient Feedback Strategy (CCFS) [14,22] and Prediction Feedback Strategy (PFS) [14,23]. It has been proved that MVFS is more efficient than TTFS which brings in a lag effect making it impossible to provide road users with the real situation for each route [21] and CCFS is more efficient than MVFS because the random brake mechanism of the Nagel–Schreckenberg (NS) model [12] yields fragile stability of the velocity [22]. However, CCFS is still not the best one, due to the fact that its feedback is not in time, so it cannot reflect the real road situation immediately. Compared with CCFS, PFS can provide road users with better guidance because it can predict the future condition of the road. However, we never see these advanced feedback strategies applied in a multi-route system in the former work. In this paper, we first report simulation results, adopting four different feedback strategies on a three-route scenario with single route following the NS mechanism. We will also discuss the situation of multi-route systems at the end of this paper.

The paper is arranged as follows: In Section 2, the NS model and a three-route scenario are briefly introduced, together with the four feedback strategies of TTFS, MVFS, CCFS and PFS all depicted in more detail. In Section 3, some simulation

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results will be presented and discussed, based on the comparison of four different feedback strategies. In the last section, we will draw some conclusions.

2. The model and feedback strategies

2.1. The NS mechanism

The Nagel–Schreckenberg (NS) model is so far the most popular and simplest cellular automaton model used in analyzing traffic flow [1–3,12], where the one-dimensional CA with periodic boundary conditions is used to investigate highway and urban traffic. This model can reproduce the basic features of real traffic like stop-and-go waves, phantom jams, and the phase transition on a fundamental diagram. In this section, the NS mechanism will be briefly introduced as a basis of analysis.

The road is subdivided into cells with a length of $\Delta x = 7.5$ m. Let N be the total number of vehicles on a single route of length L; then the vehicle density is $\rho = N/L$. $g_n(t)$ is defined to be the number of empty sites in front of the nth vehicle at time t, and $v_n(t)$ to be the speed of the nth vehicle, i.e., the number of sites that the nth vehicle moves during the time step t. In the NS model, the maximum speed is fixed at $v_{\max} = M$. In the present paper, we set $M = 3$ for simplicity.

The NS mechanism can be decomposed into the following four rules (parallel dynamics):

Rule 1. Acceleration: $v_i \leftarrow \min(v_i + 1, M)$.

Rule 2. Deceleration: $v_i' \leftarrow \min(v_i, g_i)$.

Rule 3. Random brake: with a certain brake probability p do $v_i'' \leftarrow \max(v_i' - 1, 0)$.

Rule 4. Movement: $x_i \leftarrow x_i + v_i''$.

The fundamental diagram characterizes the basic properties of the NS model which has two regimes called the "free-flow" phase and "jammed" phase. The critical density, basically depending on the random brake probability p, divides the fundamental diagram into these two phases.

2.2. The three-route scenario

The three-route model, in which road users choose one of the three routes according to real-time information feedback, is similar to the two-route model. The rules at the exit of the three-route system, however, are more complex than those of the two-route system, and we will explain the reason in Section 2.3. In a three-route scenario, it is supposed that there are three routes A, B and C of the same length L. At each time step, a new vehicle is generated at the entrance of the system and will choose one route. If a vehicle enters one of the three routes, its motion will follow the dynamics of the NS model. We remark that, if a new vehicle is not able to enter the desired route, it will be deleted. And a vehicle will also be removed after it reaches the end point.

Additionally, two kinds of vehicles are introduced: dynamic and static vehicles. A so-called dynamic driver will make a choice on the basis of the information feedback [20], while a static one just enters a route at random ignoring any advice. The density of dynamic and static travelers are S_{dyn} and $1 - S_{dyn}$, respectively.

The simulations are performed in the following steps. First, we set the routes and board empty. Second, after the vehicles enter the routes, according to four different feedback strategies, information will be generated, transmitted, and displayed on the board at each time step. Finally, the dynamic road users will choose the route with better condition according to the dynamic information at the entrance of three routes.

2.3. Related definitions

The road conditions can be characterized by fluxes of three routes, and the flux of one lane is defined as follows:

$$F = V_{mean}\rho = V_{mean}\frac{N}{L} \tag{2.1}$$

where V_{mean} represents the mean velocity of all the vehicles on one of the roads, N denotes the vehicle number on each road, and L is the length of three routes. Then we describe four different feedback strategies.

TTFS: At the beginning, all routes are empty and the information of travel time on the board is set to be the same. Each driver will record the time on entering one of the routes. Once a vehicle leaves the three-route system, it will transmit its travel time on the board and at that time a new dynamic driver will choose the road with shortest time.

MVFS: Every time step, each vehicle on the routes transmits its velocity to the traffic control center which will deal with the information and display the mean velocity of vehicles on each route on the board. Road users at the entrance will choose one road with largest mean velocity.

CCFS: Every time step, each vehicle transmits its signal to satellite; then the navigation system (GPS) will handle that information and calculate the position of each vehicle which will be transmitted to the traffic control center. The work of the traffic control center is to compute the congestion coefficient of each road and display it on the board. Road users at the entrance will choose one road with the smallest congestion coefficient.

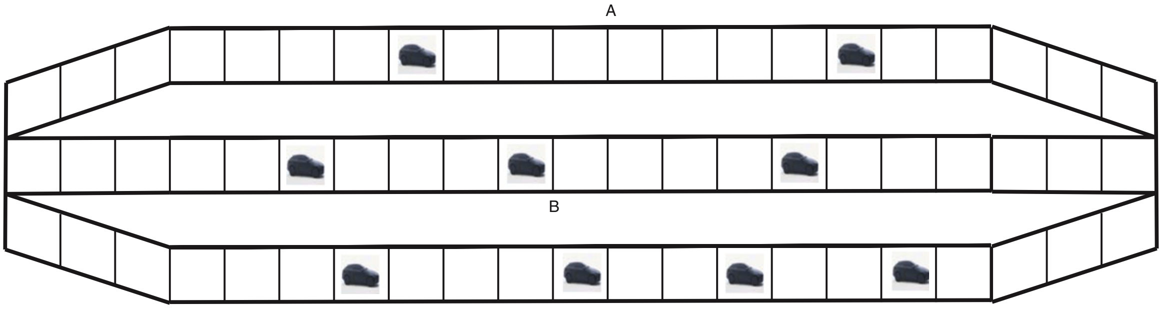

Fig. 1. The three-route system only has one entrance and one exit.

The congestion coefficient is defined as

$$C = \sum_{i=1}^{m} n_i^w. \tag{2.2}$$

Here, n_i stands for vehicle number of the ith congestion cluster in which cars are close to each other without a gap between any two of them. Every cluster is evaluated by a weight w, here $w = 2$ [22].

PFS: It is based on CCFS because CCFS is the best one among the three strategies above.

Every time step, the traffic control center will receive data from the navigation system (GPS) like CCFS. The work of the center is to compute the congestion coefficient of each lane and simulate the future road situation based on the current road situation adopting the CCFS and display the prediction congestion coefficient on the board. Road users at the entrance will choose one road with the smallest prediction congestion coefficient. For example, if the prediction time (T_p) is 50 s and the current time is the 100th second, the traffic control center will simulate the road situation at the next 50 s using CCFS, predict the road situation at the 150th second, and show the result on the board at the entrance of the road. Finally the road users at the 100th second will choose one lane with the smallest prediction congestion coefficient at the 150th second predicted by the new strategy. So by analogy, the road user at the entrance at the 101th second will choose one road with the smallest prediction congestion coefficient at the 151th second predicted by this strategy as explained above and so on.

In this paper, the three-route system has only one entrance and one exit instead of one entrance and three exits as shown in Fig. 1. So the road condition in this paper is closer to reality. The rules at the exit of the three-route system are as follows:

(a) At the end of three routes, the nearest vehicle to the exit goes first.
(b) If the vehicles at the end of three routes have the same distance to the exit, the fastest one drives on, and it goes out first.
(c) If the vehicles at the end of three routes have the same speed and distance to the exit, the vehicle on the route which has most vehicles drives out first.
(d) If the rules (a), (b) and (c) are satisfied at the same time, then the vehicles go out randomly.

Though the rules in the three-route system seem to be the same as those in the two-route system, if you consider them carefully, you will see that rules in the three-route system are much more complex. For example, among the vehicles on routes A, B and C, the vehicle on route A is the nearest one to the exit and meanwhile the vehicles on routes B and C have the same distance to the exit which will never happen in the two-route system. In the following section, the performances obtained by using four different feedback strategies will be shown and discussed in more detail.

3. Simulation results

All simulation results shown here are obtained by 90 000 iterations excluding the initial 5000 time steps. From the data shown above, we can find out that the number of time steps needed to reach a stable state for a three-route system is much greater than that for a two-route system, where one only needs 25 000 time steps to reach a stable state [23]. So this brings about a lot of difficulties in our current work. Fig. 2 shows the dependence of the average flux and prediction time (T_p) adopting the prediction feedback strategy. As to the routes' processing capacity, we can see that in Fig. 2, the prediction time (T_p) corresponding to the highest value of the average flux is about 260 time steps, which is a much greater number than that before [23]. Hence, we will use $T_p = 260$ in the following paragraphs.

In contrast with that for PFS, the fluxes of the three routes adopting CCFS, MVFS and TTFS show oscillation (see Fig. 3), obviously due to the information lag effect [22]. This lag effect can be understood, because the other three strategies cannot reflect the road's current conditions. For TTFS, the travel time reported by a driver at the end of three routes only represents the road condition in front, and perhaps the vehicles behind have got into the jammed state. Unfortunately, this information will induce more vehicles to choose that driver's route until a vehicle from the jammed cluster leaves the system. This effect apparently does harm to the system. For MVFS, we have mentioned that the NS model has a random brake scenario which

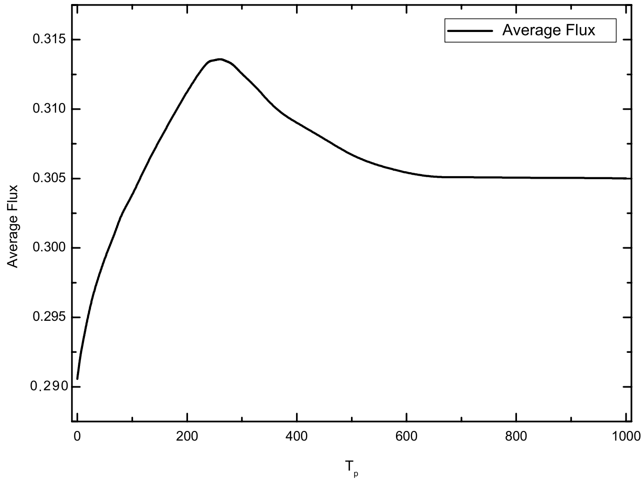

Fig. 2. Average flux *vs* prediction time (T_p). The parameters are $L = 2000$, $p = 0.25$, and $S_{dyn} = 0.5$.

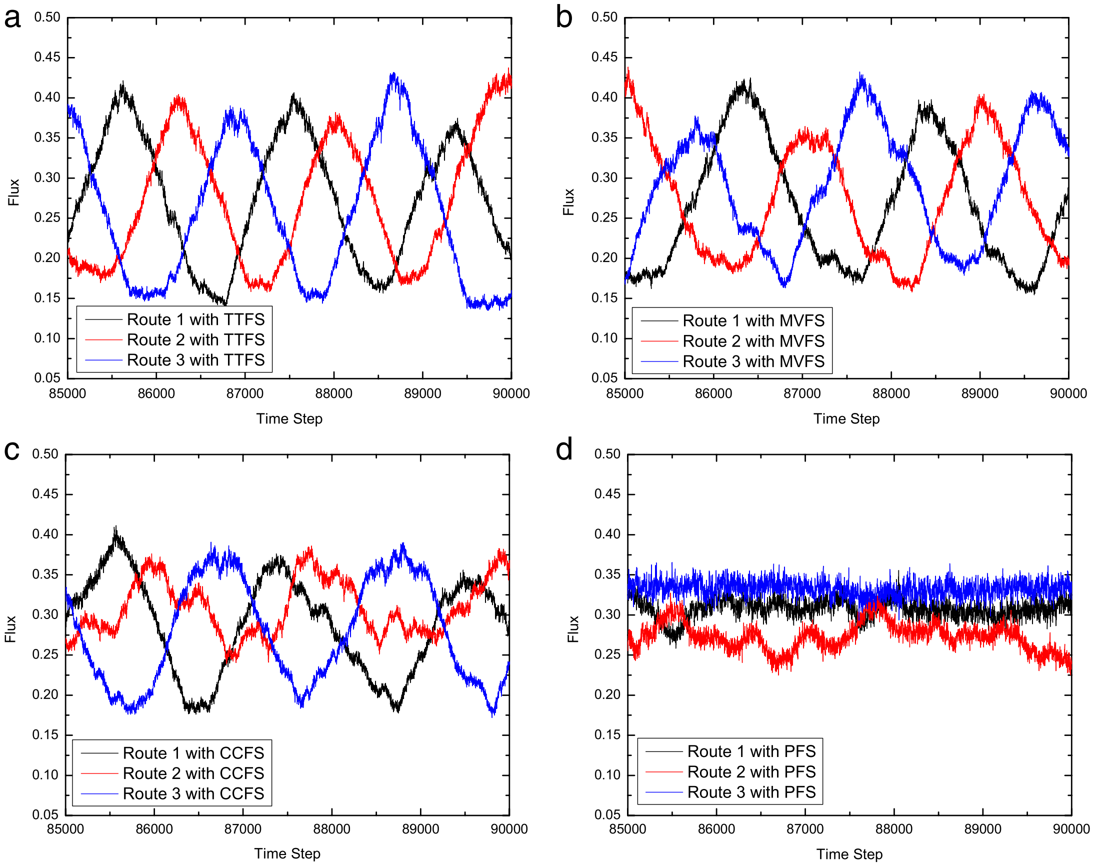

Fig. 3. (Color online) (a) Flux of each route with the travel time feedback strategy. (b) Flux of each route with the mean velocity feedback strategy. (c) Flux of each route with the congestion coefficient feedback strategy. (d) Flux of each route with the prediction feedback strategy. The parameters are $L = 2000$, $p = 0.25$, $S_{dyn} = 0.5$, and $T_p = 260$.

causes fragile stability of the velocity, so MVFS cannot completely reflect the real conditions of routes. The other reason for the disadvantage of MVFS is that the flux consists of two parts, mean velocity and vehicle density, but MVFS only grasps one part and lacks the other part of the flux. Another reason for the oscillation of the three former strategies is that the three-route system only has one exit; therefore, only one car can go out at each time step, which may result in traffic jams happening at the end of the routes. However, the new strategy can predict the effects on the route conditions caused by

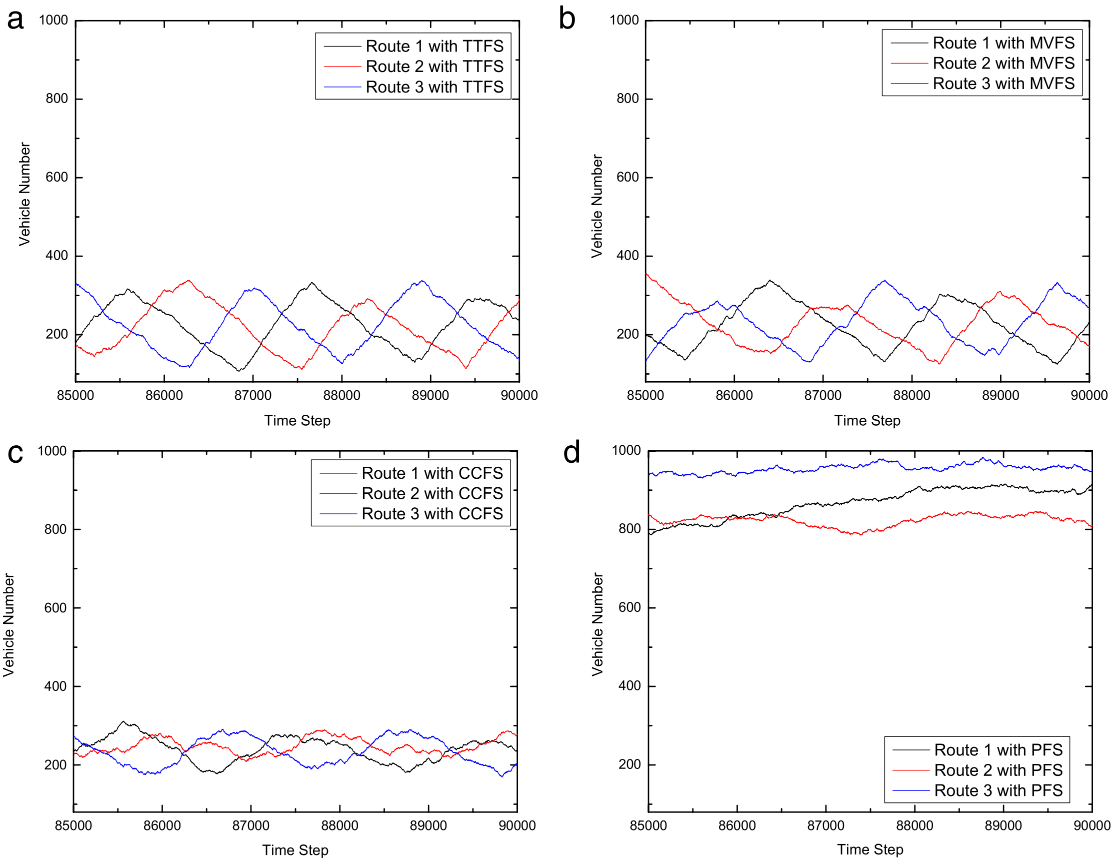

Fig. 4. (Color online) (a) Vehicle number for each route with the travel time feedback strategy. (b) Vehicle number for each route with the mean velocity feedback strategy. (c) Vehicle number for each route with the congestion coefficient feedback strategy. (d) Vehicle number for each route with the prediction feedback strategy. The parameters are set the same as in Fig. 3.

the traffic jams happening at the end of the routes, then try to avoid the traffic jams happening to the best of its ability and alleviate the negative effects as much as possible. Here, we want to stress that though PFS tries to avoid the jammed state, the structure of the traffic system (one exit) still makes jams possible at the end of the route occasionally; also this can explain the slight oscillation in Fig. 3(d). Hence, the new strategy may greatly improve the road situation. Compared with CCFS, the performance adopting PFS is remarkably improved, not only for the value but also the stability of the flux. Hence, as regards the flux of the three-route system, PFS is the best.

In Fig. 4, vehicle number versus time step shows almost the same tendency as Fig. 3, and that the routes' accommodating capacity is greatly enhanced, with an increase in average vehicle number from 230 to 870, so perhaps the high fluxes of the three routes with PFS are mainly due to the increase in vehicle numbers. Here, someone may ask whether a high vehicle number will result in a jammed cluster in each route. As to the routes' stability, we know that PFS is the best (see Figs. 3 and 4), which means that the vehicles should be almost uniformly distributed on each route instead of being together at the end of the routes. Furthermore, even though there are 870 vehicles on each route, most vehicles can still occupy two sites on each lane and a few vehicles may even occupy three route sites because the total length of each route is fixed at 2000 sites. Meanwhile, this means that there is only one site between most of the vehicles. Though the vehicles are almost distributed separately on each lane, the one-exit structure means that jams still have a chance of happening at the end of the route; however, PFS can prevent jams from expanding further and alleviate the negative effects as much as possible, so the jammed state will soon disappear. So by analogy, even if the jams happen again, the poor road condition will be relieved in a short time. Hence, there is some connection between the high accommodating capacity shown in Fig. 4 and the traffic jams discussed in the paragraph above.

In Fig. 5, speed versus time step shows that although the speed is the most stable on using the prediction feedback strategy, it is the lowest among the four different strategies. The reason is that the speed partially depends on the number of empty sites between two vehicles on the lane. As mentioned before, the routes' accommodating capacity is the best on using PFS, indicating that the speed adopting PFS is the lowest. From the stability of the velocity, we infer that the vehicles should drive at almost uniform speeds on each route. Without considering other factors, the speed should be a little more than 1 because there is only one site between most of the vehicles as mentioned above and the vehicle behind another vehicle

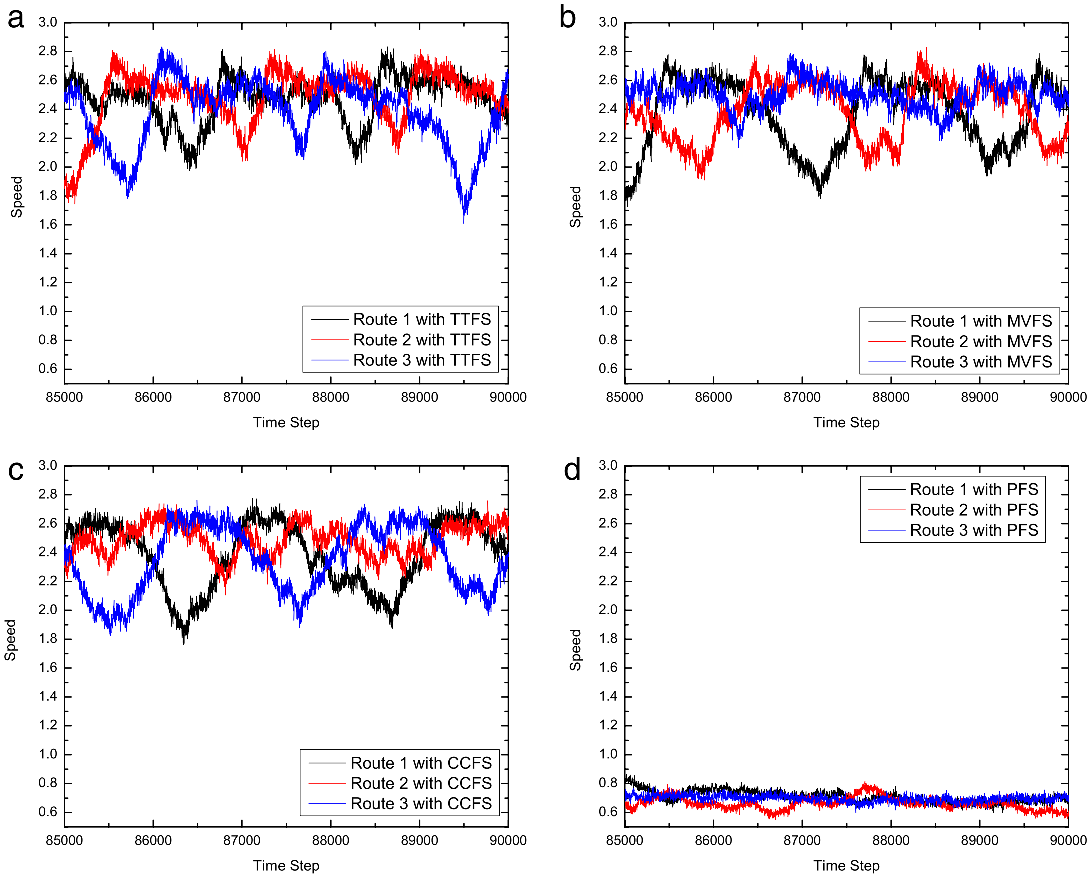

Fig. 5. (Color online) (a) Average speed on each route with the travel time feedback strategy. (b) Average speed on each route with the mean velocity feedback strategy. (c) Average speed on each route with the congestion coefficient feedback strategy. (d) Average speed on each route with the prediction feedback strategy. The parameters are set the same as in Fig. 3.

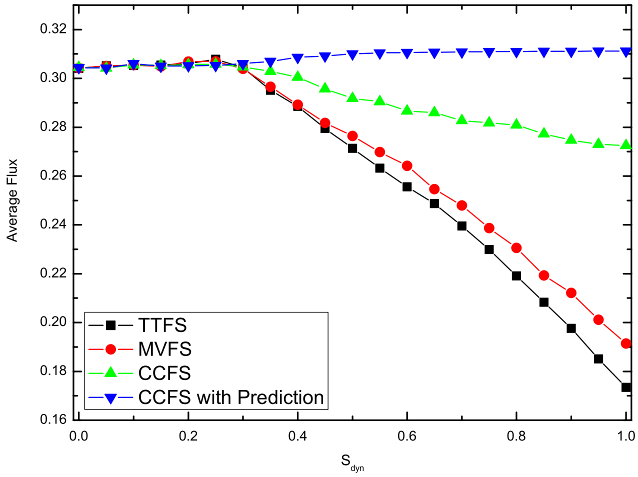

Fig. 6. (Color online) Average flux from using different strategies *vs* S_{dyn} in the three-route system; L is fixed at 2000, p is fixed at 0.25 and T_p is fixed at 260.

can move at most over the current empty sites between them, which is also required by the NS mechanism. If we take the random brake effects and the occasional jams at the end of the route into account, the vehicles' average velocity being lower than 1 is possible and reasonable; therefore, the average velocity $V_{avg} \sim 0.7$ in this paper could be understood and there should be no conflicts between the average velocity and the almost uniform distribution of vehicles on each route. The other three strategies' high speeds may result from the vehicles at the beginning or middle parts of the lane which

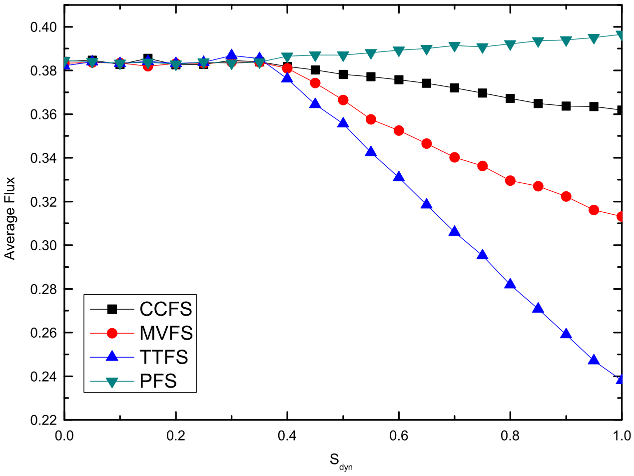

Fig. 7. (Color online) Average flux from using different strategies *vs* S_{dyn} in the two-route system; L is fixed at 2000, p is fixed at 0.25 and T_p is fixed at 60.

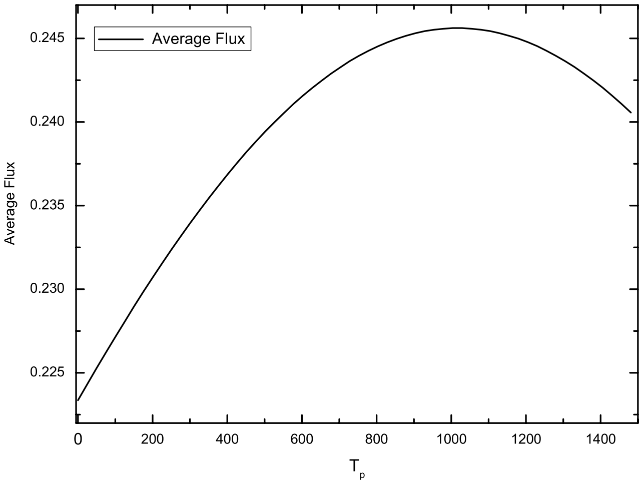

Fig. 8. Average flux *vs* prediction time (T_p) in a four-route model. The parameters are $L = 2000$, $p = 0.25$, and $S_{dyn} = 0.5$.

have high velocities instead of the vehicles near the exit, because the average velocity in each lane depends on all vehicles' velocities. Fortunately, the flux consists of two parts, mean velocity and vehicle density. Therefore, as long as the vehicle number (because the vehicle density is $\rho = N/L$, and the L is fixed at 2000, so $\rho \propto$ vehicle number (N)) is large enough, the flux can also be the largest.

Fig. 6 shows that the average flux fluctuates feebly with a persisting increase of dynamic travelers on using four different strategies. As to the routes' processing capacity, the prediction feedback strategy is proved to be the best one because the flux is always the largest at each S_{dyn} value and is enhanced with a persisting increase of dynamic travelers. A question here for some readers is why the average fluxes in Fig. 6 adopting four different strategies are smaller than that (see Fig. 7) shown in the former work [23] where the traffic system has only two routes and the prediction time (T_p) is fixed at 60. The reason is that the three-route system in this paper still permits at most one car to enter the entrance at each time step. Hence, the more routes the traffic system has, the lower average flux it has.

4. Conclusion

We obtain the simulation results from applying four different feedback strategies, i.e., TTFS, MVFS, CCFS and PFS, on a three-route scenario all with respect to flux, number of vehicles, speed, average flux versus T_p and average flux versus S_{dyn}. The results indicate that PFS has more advantages than the other three strategies in the three-route system which has only one entrance and one exit. We also find out that it will take much more time to reach the stable state in the three-route system than in the two-route system. In contrast with the other three feedback strategies, PFS can significantly improve the road condition, including increasing the vehicle number and flux, reducing oscillation, and that the average flux is enhanced with increase of S_{dyn}. And this can be understood because PFS can eliminate the lag effect. The numerical simulations demonstrate that the prediction time (T_p) plays a very important role in improving the road situation.

We also perform a simulation of average flux versus T_p on a four-route scenario (see Fig. 8) which is obtained by 90 000 iterations excluding the initial 5000 time steps. We can see that in Fig. 8 the prediction time (T_p) corresponding to the highest value of the average flux is about 1020 time steps, which is far greater than for the two-route and three-route systems. Here, we can make a reasonable assumption that if one car passes along the route with the average speed $v_{mean} \sim 2$, then the time in which it passes the route is $T_{pass} \sim 1000$ because the total length of one route is $L = 2000$. If T_{pass} and T_p are of the same order of magnitude ($T_{pass} \sim T_p$), then the prediction feedback strategy will become invalid because the car at the entrance of the route at first will leave the traffic system after T_{pass}. So we come to the conclusion that the prediction feedback strategy is appropriate in multi-route systems when the length of the route is long enough to ensure $T_{pass} > T_p$.

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