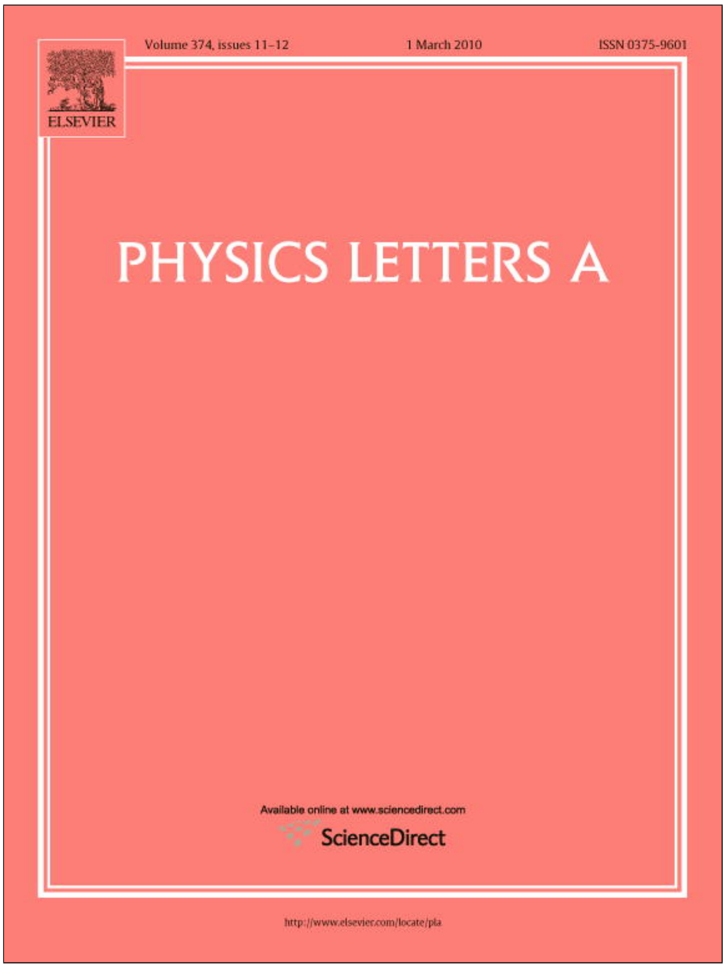
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Weighted congestion coefficient feedback in intelligent transportation systems

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ABSTRACT

In traffic systems, a reasonable information feedback can improve road capacity. In this Letter, we study dynamics of traffic flow with real-time information. And the influence of a feedback strategy named Weighted Congestion Coefficient Feedback Strategy (WCCFS) is introduced, based on a two-route scenario in which dynamic information can be generated and displayed on the board to guide road users to make a choice. Our model incorporates the effects of adaptability into the cellular automaton models of traffic flow and simulation results adopting this optimal information feedback strategy have demonstrated high efficiency in controlling spatial distribution of traffic patterns compared with the other three information feedback strategies, i.e., vehicle number and flux.

1. Introduction

Traffic flow, a kind of multi-body system consisting of interacting vehicles, shows various complex behaviors. Therefore, the traffic problems have attracted the interests of many physicists [1–3] and also a lot of theories have been proposed such as car-following theory [4,5], kinetic theory [6–13] and particle-hopping theory [14–16]. These theories have the advantages of alleviating the traffic congestion and enhancing the capacity of existing infrastructure. Although dynamics of traffic flow with real-time traffic information have been extensively investigated [17–22], finding a more efficient feedback strategy is still an overall task. Recently, some real-time feedback strategies have been put forward, such as Travel Time Feedback Strategy (TTFS) [17,23], Mean Velocity Feedback Strategy (MVFS) [17,24] and Congestion Coefficient Feedback Strategy (CCFS) [17,25]. It has been proved that MVFS is more efficient than TTFS which brings a lag effect to make it impossible to provide the road users with the real situation of each route [24] and CCFS is more efficient than MVFS because the random brake mechanism of the Nagel–Schreckenberg (NS) model [14] brings fragile stability of velocity [25]. However, CCFS is still not the best one due to the fact that it still doesn't take the structure of the route into consideration. Therefore, it cannot reflect the weights of different parts of the lane and some other reasons will be discussed delicately in this Letter. In order to provide road users with better guidance, a strategy named Weighted Congestion Coefficient Feedback Strategy (WCCFS) is presented. We report the simulation results adopting four different feedback strategies in a two-route scenario with a single route following the NS mechanism.

The Letter is arranged as following: In Section 2 the NS model and a two-route scenario are briefly introduced, together with four feedback strategies of TTFS, MVFS, CCFS and WCCFS all depicted in more details. In Section 3 some simulation results will be presented and discussed based on the comparison of four different feedback strategies. In the last section, we will make some conclusions.

2. The model and feedback strategies

2.1. NS mechanism

The Nagel–Schreckenberg (NS) model is so far the most popular and simplest cellular automaton model in analyzing the traffic flow [1–3,14,26,27], where the one-dimensional CA with periodic boundary conditions is used to investigate highway and urban traffic. This model can reproduce the basic features of real traffic like stop-and-go wave, phantom jams, and the phase transition on a fundamental diagram. In this section, the NS mechanism will be briefly introduced as a base of analysis.

The road is subdivided into cells with a length of $\Delta x = 7.5$ m. Let N be the total number of vehicles on a single route of length L, then the vehicle density is $\rho = N/L$. $g_n(t)$ is defined to be the number of empty sites in front of the nth vehicle at time t, and $v_n(t)$ to be the speed of the nth vehicle, i.e., the number of sites

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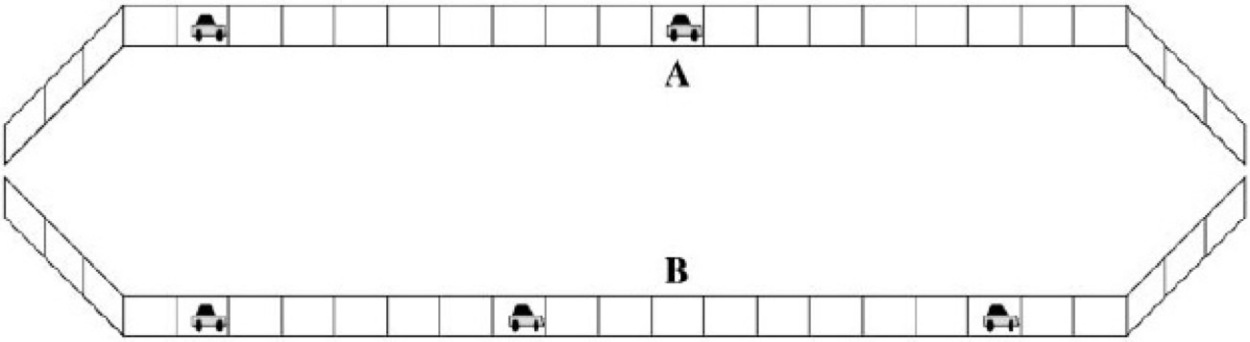


Fig. 1. The two-route system only has one entrance and one exit which is different from the road situation in former work.

that the nth vehicle moves during the time step t. In the NS model, the maximum speed is fixed to be $v_{\max} = M$. In the present Letter, we set $M = 3$ for simplicity.

The NS mechanism can be decomposed to the following four rules (parallel dynamics):

Rule 1. Acceleration: $v_i \leftarrow \min(v_i + 1, M)$;
Rule 2. Deceleration: $v_i' \leftarrow \min(v_i, g_i)$;
Rule 3. Random brake: with a certain brake probability p do $v_i'' \leftarrow \max(v_i' - 1, 0)$; and
Rule 4. Movement: $x_i \leftarrow x_i + v_i''$.

The fundamental diagram characterizes the basic properties of the NS model which has two regimes called "free-flow" phase and "jammed" phase. The critical density, basically depending on the random brake probability p, divides the fundamental diagram to these two phases.

2.2. Two-route scenario

Wahle et al. [23] first investigated the two-route model in which road users choose one of the two routes according to the real-time information feedback. In a two-route scenario, it is supposed that there are two routes A and B of the same length L. At every time step, a new vehicle is generated at the entrance of two routes and will choose one route. If a vehicle enters one of two routes, the motion of it will follow the dynamics of the NS model. As a remark, if a new vehicle is unable to enter the desired route, it will be deleted. And a vehicle will also be removed after it reaches the end point.

Additionally, two types of vehicles are introduced: dynamic and static vehicles. If a driver is a so-called dynamic one, he will make a choice on the basis of the information feedback [23], while a static one just enters a route at random ignoring any advice. The density of dynamic and static travelers are S_{dyn} and $1 - S_{dyn}$, respectively.

The simulations are performed by the following steps: first, we set the routes and board empty; second, after the vehicles enter the routes, according to four different feedback strategies, information will be generated, transmitted, and displayed on the board at each time step. Finally, the dynamic road users will choose the route with better condition according to the dynamic information at the entrance of two routes.

2.3. Related definitions

The road conditions can be characterized by fluxes of two routes, and flux is defined as follows:

$$F = V_{mean}\rho = V_{mean}\frac{N}{L} \tag{2.1}$$

where V_{mean} represents the mean velocity of all the vehicles on one of the roads, N denotes the vehicle number on each road, and L is the length of two routes. Then we describe four different feedback strategies, respectively.

TTFS: At the beginning, both routes are empty and the information of travel time on the board is set to be the same. Each driver will record the time when he enters one of the routes. Once a vehicle leaves the two-route system, it will transmit its travel time on the board and at that time a new dynamic driver will choose the road with shorter time.

MVFS: Every time step, each vehicle on the routes transmits its velocity to the traffic control center which will deal with the information and display the mean velocity of vehicles on each route on the board. Road users at the entrance will choose one road with larger mean velocity.

CCFS: Every time step, each vehicle transmits its signal to satellite, then the navigation system (GPS) will handle that information and calculate the position of each vehicle which will be transmitted to the traffic control center. The work of the traffic control center is to compute the congestion coefficient of each road and display it on the board. Road users at the entrance will choose one road with smaller congestion coefficient.

The congestion coefficient is defined as

$$C = \sum_{i=1}^{p} n_i^w. \tag{2.2}$$

Here, n_i stands for vehicle number of the ith congestion cluster in which cars are close to each other without a gap between any two of them. Every cluster is evaluated a weight w, here $w = 2$ [25].

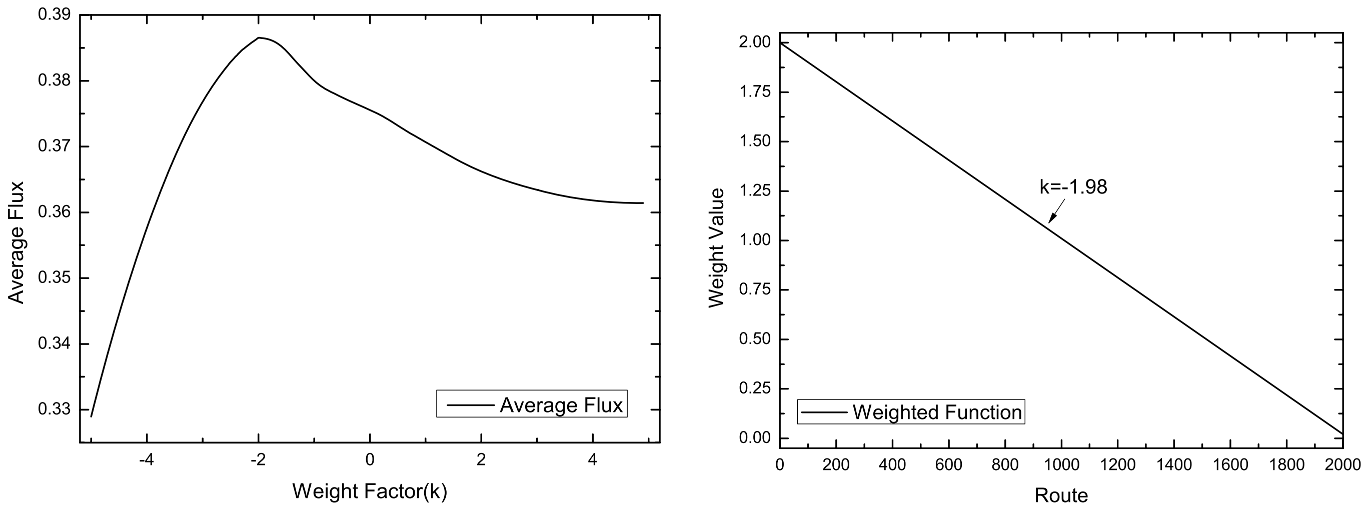
WCCFS: WCCFS is based on CCFS, because CCFS is the best one among the three strategies above.

Every time step, the traffic control center will receive data from the navigation system (GPS) like CCFS, and the work of the center is to compute the congestion coefficient of each road with a reasonable weighted function and display it on the board. Road users at the entrance will choose one road with smaller weighted congestion coefficient.

The weighted congestion coefficient is defined as

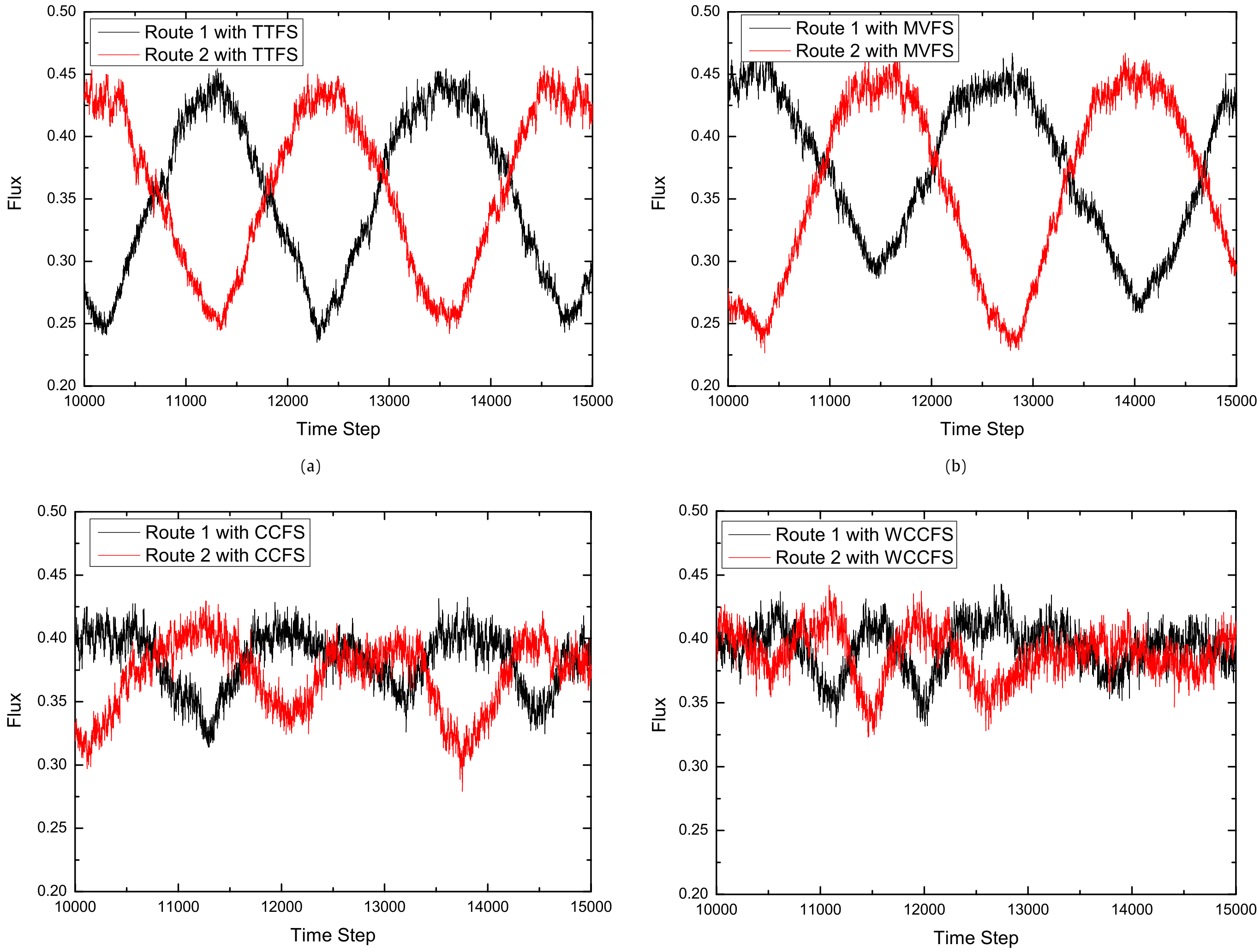
$$C_w = \sum_{i=1}^{p} F(n_m) n_i^w. \tag{2.3}$$

Here, n_i stands for vehicle number of the ith congestion cluster in which cars are close to each other without a gap between any two of them. And n_m stands for the position of the ith congestion cluster, for example, if the vehicle number of the ith congestion cluster is odd, the position of the median of congestion cluster stands for the position of it, and if the vehicle number of the ith congestion cluster is even, then we use the result of median rounding $\lfloor n_m \rfloor$ to



(a) (b)

Fig. 2. (a) Average flux versus weight factor (k). (b) Weight value of each site of the route. The parameters are $L = 2000$, $p = 0.25$, and $S_{dyn} = 0.5$.



(a) (b) (c) (d)

Fig. 3. (Color online.) (a) Flux of each route with travel time feedback strategy. (b) Flux of each route with mean velocity feedback strategy. (c) Flux of each route with congestion coefficient feedback strategy. (d) Flux of each route with weighted congestion coefficient feedback strategy. The parameters are $L = 2000$, $p = 0.25$, $S_{dyn} = 0.5$, and weight factor (k) is fixed to be -1.98.

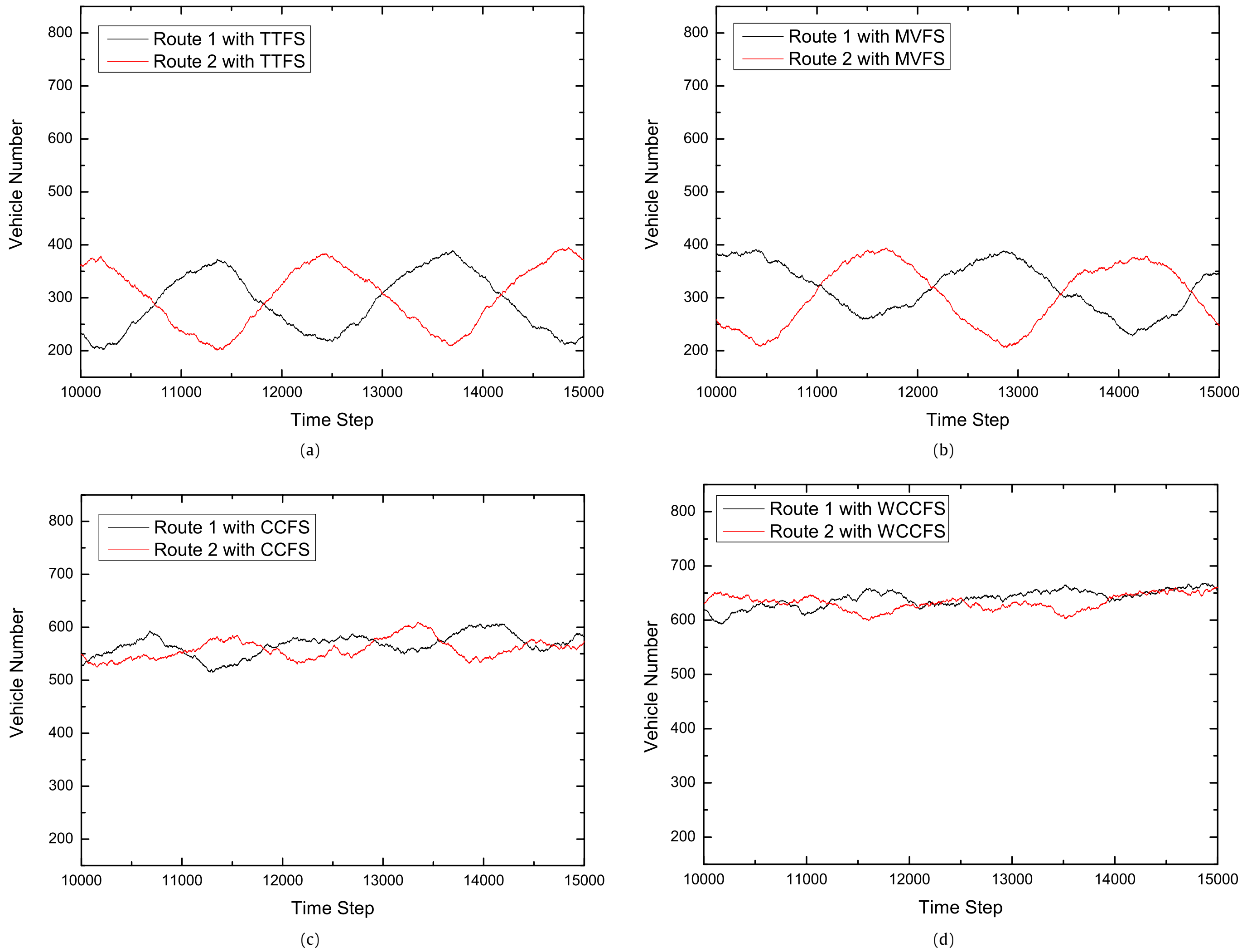


Fig. 4. (Color online.) (a) Vehicle number of each route with travel time feedback strategy. (b) Vehicle number of each route with mean velocity feedback strategy. (c) Vehicle number of each route with congestion coefficient feedback strategy. (d) Vehicle number of each route with weighted congestion coefficient feedback strategy. The parameters are set the same as in Fig. 3.

represent the position of the congestion cluster. And $F(x)$ stands for the weighted function of each route. Here we also set $w = 2$ as above.

After we try some functions such as $F(x) = \mathrm{Cos}(ax) + b$, we find $F(x) = kx + b$ is the best one in terms of improving the capacity of the road. Here, we set $b \neq 0$ for the reason that it will cause the absolute weight value of the first route site always to be the smallest when $b = 0$. And in this Letter, we set $b = 2.0$. Then we get the function as follows:

$$F(x) = k \times x + b = k \times \frac{n_m}{2000} + 2.0. \tag{2.4}$$

Finally the expression of C_w becomes

$$C_w = \sum_{i=1}^{p} F(n_m) n_i^w = \sum_{i=1}^{p} \left(k \times \frac{n_m}{2000} + 2.0 \right) \times n_i^2. \tag{2.5}$$

We also find how efficient the new strategy to improve the road capacity depends on the vale of weight factor (slope-k) which we will discuss carefully in Section 3.

Compared with the former work [23–25], another important difference in this Letter is that we set the two-route system to have only one entrance and one exit as shown in Fig. 1 while the former two-route system has one entrance and two exits. So we do research work based on the two-route system which is closer to the reality instead of simply repeating the former work. The rules at the exit of the two-route system are as follows:

(a) At the end of two routes, the car that is nearer to the exit goes first.
(b) If the cars at the end of two routes have the same distance to the exit, faster one drives, first it goes out.
(c) If the cars at the end of two routes have the same speed and distance to the exit, the car in the route which owns more cars drives out first.
(d) If the rules (a), (b) and (c) are satisfied at the same time, then the cars go out randomly.

In the following section, performance by using four different feedback strategies will be shown and discussed in more details.

3. Simulation results

All simulation results shown here are obtained by 15 000 iterations excluding the initial 5000 time steps. Fig. 2(a) shows the dependence of average flux and weight factor (k) by using the new strategy. As to the routes' processing capacity, we can see that in Fig. 2(a) there is a positive peak structure at the vicinity of

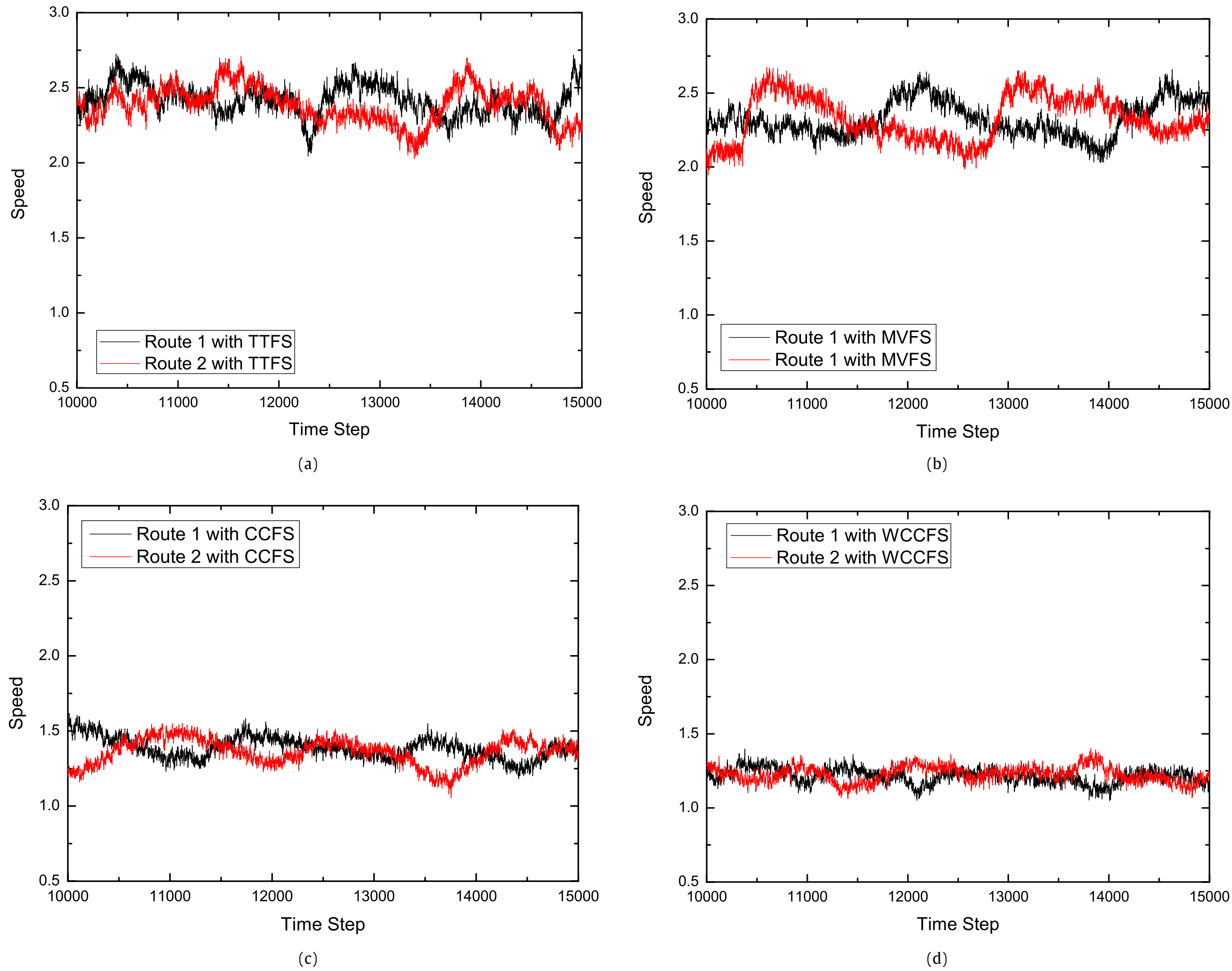


Fig. 5. (Color online.) (a) Average speed of each route with travel time feedback strategy. (b) Average speed of each route with mean velocity feedback strategy. (c) Average speed of each route with congestion coefficient feedback strategy. (d) Average speed of each route with weighted congestion coefficient feedback strategy. The parameters are set the same as in Fig. 3.

$k \sim -1.98$. So we will use $k = -1.98$ in the following paragraphs. In Fig. 2(b), we present the weight value of each route site. You will find the weight value of the entrance is much larger than that of the exit when adopting WCCFS. There are several reasons for this result. First, the road users use the information on the board at the entrance of the traffic system to decide which route to enter. And this will directly affect the road situation. For instance, vehicles entering the route with larger congestion coefficient will definitely cause the road situation worse than before. Second, the small weight value at the end of the route will alleviate the negative effects of congestion caused by the traffic jam which will be explained in the next paragraph.

In contrast with WCCFS, the fluxes of two routes adopting CCFS, MVFS and TTFS show larger oscillation (see Fig. 3). This oscillation effect can be understood for several reasons. On one hand, the other three strategies cannot reflect the weights of different parts of the lane. One the other hand, the two-route system only has one exit, therefore, only one car can go out at each time step. And this may result in the traffic jam to happen at the end of the routes. However, the new strategy can make the value of congestion coefficient at the end of the routes smaller than before as shown in Fig. 2(b) which is equivalent to alleviate the negative effects of congestion caused by the traffic jam. Hence, the new strategy may improve the road situation. Compared to CCFS, the performance adopting WCCFS is remarkably improved, not only on the value but also the stability of the flux. Therefore, as to the flux of the two-route system, WCCFS is the best one.

In Fig. 4, vehicle number versus time step shows almost the same tendency as Fig. 3, and that the routes' accommodating capacity is greatly enhanced with an increase in average vehicle number from 295 to 625, so perhaps the high fluxes of two routes with WCCFS are mainly due to the increase of vehicle number. A question here for some readers is why the vehicle number in Fig. 4 using other three strategies is larger than the figures shown in the former work [25]. The reason is that the road situation is different from the work before. The two-route system in this Letter only has one exit, therefore, only one car can go out at each time step which will lead to the increase of vehicle number in each route.

In Fig. 5, speed versus time step shows that although the speed is the stablest by using the new strategy, it is the lowest among the four different strategies. The reason is that the routes' accommodating capacity is the best by using the new strategy. And as mentioned before, the road has only one exit and only one car can go out at each time step. Therefore, more cars the lane owns, lower speeds the vehicles have. Fortunately, flux consists of two

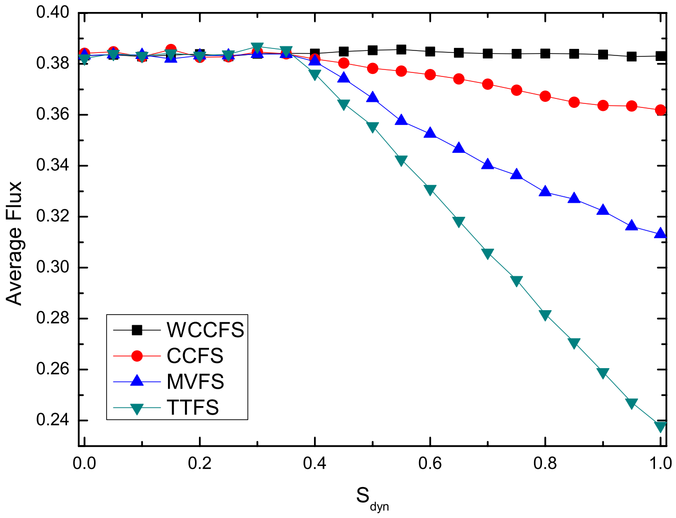


Fig. 6. (Color online.) Average flux by performing different strategy versus S_{dyn}; L is fixed to be 2000, p is fixed to be 0.25 and weight factor (k) is fixed to be -1.98.

parts, mean velocity and vehicle density. Hence, as long as the vehicle number (because the vehicle density is $\rho = N/L$, and the L is fixed to be 2000, so $\rho \propto$ vehicle number (N)) is large enough, the flux can also be the largest.

Fig. 6 shows that the average flux fluctuates feebly with a persisting increase of dynamic travelers by using four different strategies. As to the routes' processing capacity, the new strategy is proved to be the best one because the flux is always the largest at each S_{dyn} value and keeps the two routes' fluxes in balance. Maybe someone will ask why the value of average flux in Fig. 6 adopting other three strategies is smaller than the results shown in former work [25]. This may also result from the different structure of the traffic system which has only one exit as explained above.

4. Conclusion

We obtain the simulation results of applying four different feedback strategies, i.e., TTFS, MVFS, CCFS and WCCFS on a two-route scenario all with respect to flux, number of cars, speed, average flux versus weight factor (k) and average flux versus S_{dyn}. The results indicate that WCCFS has more advantages than the other three strategies in the two-route system with only one entrance and one exit. The highlight of this Letter is that it brings forward a new quantity namely weight factor (k) to radically improve the road conditions. In contrast with other three feedback strategies, WCCFS can significantly improve the road conditions, including increasing vehicle number and flux, reducing oscillation, and that average flux is stable with the increase of S_{dyn}. And it can be understood because the new strategy can reflect the weights of different parts of the route and alleviate the negative effects of congestion caused by the traffic jam at the end of the route. The numerical simulations demonstrate that the weight factor (k) plays a very important role in improving the road situation.

Due to the rapid development of modern scientific technology, it is not difficult to realize WCCFS in reality. If only a navigation system (GPS) is installed in each vehicle, thus the position information of vehicles will be known. Then WCCFS can come true through computational simulation by acting the weight value on each congestion cluster on the basis of CCFS. Also, it will cost no more than CCFS because the computers using to compute the congestion coefficient can also calculate the weighted congestion coefficient. Taking into account the reasonable cost and more accurate description of road condition, we think this new feedback strategy shall be applicable.

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