International Journal of Modern Physics C
Vol. 21, No. 8 (2010) 1081–1093

DOI: 10.1142/S0129183110015683

EFFECTS OF VEHICLE NUMBER FEEDBACK IN MULTI-ROUTE INTELLIGENT TRAFFIC SYSTEMS

CHUANFEI DONG

School of Earth and Atmospheric Sciences
Georgia Institute of Technology
Atlanta GA 30332, United States

Department of Modern Physics and Nonlinear Science Center
University of Science and Technology of China
Hefei, Anhui 230026, P. R. China
dcfy@gatech.edu

XU MA

Department of Physics, Syracuse University
Syracuse NY 13244, United States

Department of Modern Physics and Nonlinear Science Center
University of Science and Technology of China
Hefei, Anhui 230026, P. R. China

BINGHONG WANG

Department of Modern Physics and Nonlinear Science Center
University of Science and Technology of China
Hefei, Anhui 230026, P. R. China

The Research Center for Complex System Science
University of Shanghai for Science and Technology
and Shanghai Academy of System Science
Shanghai 200093, P. R. China

Received 24 October 2009
Accepted 30 July 2010

We first study dynamics of traffic flow with real-time information and the influence of a new feedback strategy named Vehicle Number Feedback Strategy (VNFS) in a multi-route scenario in which dynamic information can be generated and displayed on the board (the board refers to a variable message sign where information on the routes is displayed) to guide road users to make a choice. In a multi-route scenario, our model incorporates the effects of adaptability into the cellular automaton models of traffic flow and simulation results adopting vehicle number feedback strategy have demonstrated high efficiency in controlling spatial distribution of traffic patterns compared with the other three information feedback strategies, i.e. Travel Time Feedback Strategy (TTFS), Mean Velocity Feedback Strategy (MVFS) and Congestion Coefficient Feedback Strategy (CCFS). We also discuss the influence of expected arrival rate (V_p) at the entrance on the average flux of each route, and we find that the flux adopting VNFS is always the largest at each V_p value among these four feedback strategies.

Keywords: Vehicle number feedback; multi-route; intelligent traffic systems; cellular automaton model; road capacity.

PACS Nos.: 89.40.Bb, 89.20.-a, 02.60.Cb.

1. Introduction

Vehicular traffic flow and related problems have triggered great interest of a community of physicists in recent years because of its various complex behaviors.[1–3] A lot of theories have been proposed such as car-following theory,[4] kinetic theory,[5–11] and particle-hopping theory.[12,13] These theories provide insights that help traffic engineers and other professionals to better manage congestion. Therefore, these theories indirectly make contributions to alleviating the traffic congestion and enhancing the capacity of existing infrastructure. Although dynamics of traffic flow with real-time traffic information have been extensively investigated,[14–19] finding a more efficient feedback strategy is still an overall task. Recently, some real-time feedback strategies have been proposed, such as Travel Time Feedback Strategy (TTFS),[14,20] Mean Velocity Feedback Strategy (MVFS),[14,21] and Congestion Coefficient Feedback Strategy (CCFS).[14,22] It has been proved that MVFS is more efficient than TTFS that brings a lag effect, making it impossible to provide the road users with the real situation of each route.[21] Furthermore, CCFS is more efficient than MVFS because the random brake mechanism of the Nagel–Schreckenberg (NS) model[12] brings about fragile stability of velocity.[22] However, CCFS is still not the best one due to the fact that it cannot reflect the actual number of vehicles on each route and some other reasons will be further discussed in the following sections. In order to provide road users with better guidance, a new information feedback strategy named vehicle number feedback strategy (VNFS) is presented. Additionally, we never see these advanced feedback strategies applied on a multi-route system in the former work. In this paper, we first investigate the effects of all these four strategies applied on a three-route model and report the simulation results with a single route following the NS mechanism.

The remainder of this paper is arranged as follows: In Sec. 2, the NS model and a three-route scenario are briefly introduced, together with four feedback strategies of TTFS, MVFS, CCFS, and VNFS all depicted in detail. In Sec. 3, some simulation results will be presented and discussed based on the comparison of four different feedback strategies. In the last section, we will make some conclusions.

2. The Model and Feedback Strategies

2.1. *NS mechanism*

The Nagel–Schreckenberg (NS) model is so far the most popular and simplest cellular automaton (CA) model in analyzing the traffic flow,[1–3,12,23–26] where the one-dimension CA with periodic boundary conditions is used to investigate highway and urban traffic. This model can reproduce the basic features of real traffic like stop-and-go wave, phantom jams, and the phase transition on a fundamental diagram. In this section, the NS mechanism will be briefly introduced as a basis of analysis.

The road is subdivided into cells with a length of $\Delta x = 7.5$ m. We set $L = 2000$ cells (corresponding to 15 km). Let N be the total number of vehicles on a single route of length L, then the vehicle density is $\rho = N/L$. A time step corresponds to $\Delta t = 1$ s, the typical time a driver needs to react. Let $g_n(t)$ denote the number of empty sites in front of the nth vehicle at time t, and $v_n(t)$ denote the speed of the nth vehicle, i.e. the number of sites that the nth vehicle moves during the time step t. In the NS model, the maximum speed is fixed at $v_{\max} = M$. In the present paper, we set $M = 3$ (corresponding to 81 km/h) for simplicity.

The NS mechanism can be decomposed into the following four rules (parallel dynamics):

Rule 1. Acceleration: $v_i \leftarrow \min(v_i + 1, M)$;
Rule 2. Deceleration: $v_i' \leftarrow \min(v_i, g_i)$;
Rule 3. Random brake: with a certain brake probability p to $v_i'' \leftarrow \max(v_i' - 1, 0)$; and
Rule 4. Movement: $x_i \leftarrow x_i + v_i''$.

The fundamental diagram characterizes the basic properties of the NS model which has two regimes called "free-flow" phase and "jammed" phase. The critical density, basically depending on the random brake probability p, divides the fundamental diagram to these two phases.

2.2. *Three-route scenario*

Wahle *et al.*[20] first investigated the two-route model in which road users choose one of the two routes according to the real-time information feedback. The three-route model, in which road users choose one of the three routes according to the real-time information feedback, is similar to the two-route model. The rules at the exit of the three-route system, however, are more complex than that of the two-route system, and we will explain the reason in Part C. In a three-route scenario, it is supposed that there are three routes A, B, and C of the same length L. At each time step, a new vehicle is generated at the entrance of three routes in probability V_p, and will choose one route. Here, the definition of the expected arrival rate (V_p) at the entrance is the probability that vehicles arrive at the entrance at each time step. For example, $V_p = 1$ means there will be one vehicle arriving at the entrance at each time step and $V_p = 0.5$ means the probability vehicles arriving at the entrance at each time step is 0.5. If a vehicle enters one of the three routes, its motion will follow the dynamics of the NS model. In our simulation, a vehicle will be removed after it reaches the endpoint. It is important to note that if a vehicle cannot enter the preferred route, it will wait till the next time step rather than entering the unpreferred route.

Additionally, two types of vehicles are introduced: dynamic and static vehicles. If a driver is a so-called dynamic one, he will make a choice on the basis of the information feedback,[20] while a static one just enters a route at random ignoring any advice. The densities of dynamic and static travelers are S_{dyn} and $1 - S_{\text{dyn}}$, respectively.

The simulations are performed by the following steps: first, we set the routes and board empty; second, after the vehicles enter the routes, according to four different feedback strategies, information will be generated, transmitted, and displayed on the board at each time step. Finally, the dynamic road users will choose the route with best condition according to the dynamic information at the entrance of three routes.

2.3. *Related definitions*

The road conditions can be characterized by fluxes of three routes. Here, it assumes that each route consists of one traffic lane. Flux of the jth route is defined as follows:

$$F_j = V^j_{\text{mean}} \rho_j = V^j_{\text{mean}} \frac{N_j}{L_j} \tag{1}$$

where V^j_{mean} represents the mean velocity of all the vehicles on the jth route, N_j denotes the vehicle number on the jth route, and L_j is the length of the jth route. Thus, the definition of the average flux should be:

$$F_{\text{avg}} = \frac{\sum_j^k F_j}{k} = \sum_j^k \frac{V^j_{\text{mean}} \times N_j}{k \times L_j}\,, \tag{2}$$

where k is the total route number of the traffic system. Therefore we set $k = 3$ in this paper. Then we describe four different feedback strategies one-by-one.

TTFS: At the beginning, all routes are empty and the information of travel time on the board is set to be the same. Each driver will record the time when he enters one of the routes. Once a vehicle leaves the three-route system, it will transmit its travel time on the board and at that time a new dynamic driver will choose the road with shortest time.

MVFS: Every time step, each vehicle on the routes transmits its velocity to the traffic control center which will deal with the information and display the mean velocity of vehicles on each route on the board. Road users at the entrance will choose one road with largest mean velocity.

CCFS: Every time step, each vehicle transmits its signal to satellite, then the navigation system (GPS) will handle that information and calculate the position of each vehicle which will be transmitted to the traffic control center. The work of the traffic control center is to compute the congestion coefficient of each lane and display it on the board. Road users at the entrance will choose one road with smallest congestion coefficient.

The congestion coefficient is defined as

$$C = \sum_{i=1}^{m} n_i^w \,, \tag{3}$$

where n_i stands for vehicle number of the ith congestion cluster in which cars are close to each other without a gap between any two of them. Every cluster is evaluated by a weight w, where $w = 2$.[22] The reason for adding weight to each cluster can be explained by the fact that travel time of the last vehicle of the cluster from the entrance to the destination is obviously affected by the size of cluster. Imagine that with the increasing of cluster size, travel time of the last vehicle will be longer more and more and the correlation between cluster size and travel time of the last vehicle is nonlinear. For simplicity, an exponent w is added to the size of each cluster to be consistent with the nonlinear relationship.[22]

VNFS: We install pressure transducers at the entrance and exit of each route. The pressure transducer can calculate the number of the vehicles on each lane. Every time step, when a vehicle enters the route, the number of the vehicles shown on the board will increase by one by using pressure transducers. When it leaves the route, the number will decrease by one in the same way. Thus the number shown on the board is the total number of vehicles on each route. Road users at the entrance will choose one lane with smallest vehicle number.

In this paper, we assume that the three-route system has only one entrance and one exit instead of one entrance and three exits as shown in Fig. 1. In reality, there are different paths for drivers to choose from one place to another place. In this paper, we focus on a three-route system. Therefore, different drivers departing from the same place could choose three different paths to get to the same destination which corresponds to the "one entrance and one exit" system. Hence, compared with the previous work,[20–22] the road condition in this paper is closer to the reality. The rules at the exit of the three-route system are as follows:

(i) At the end of three routes, the vehicle nearest to the exit goes first.
(ii) If the vehicles at the end of three routes have the same distance to the exit, the fastest a vehicle drives, it goes out first.

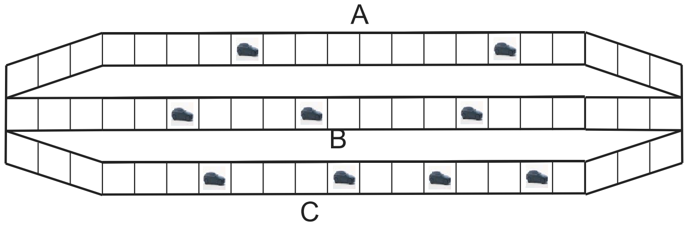

Fig. 1. The one entrance and one exit three-route system.

(iii) If the vehicles at the end of three routes have the same speed and distance to the exit, the vehicle on the route which has the most vehicles, drives out first.
(iv) If the rules (i), (ii), and (iii) are satisfied at the same time, then the vehicles go out randomly.

Though rules of the three-route system seem to be the same as that of the two-route system,[24] if you consider it carefully, you may find out that rules of the three-route system are much more complicated. For example, among the vehicles on routes A, B, and C, the vehicle on route A is the nearest one to the exit and meanwhile, the vehicles on routes B and C have the same distance to the exit which will never happen in the two-route system. In the following section, performance by using four different feedback strategies will be shown and discussed in detail.

3. Simulation Results

All simulation results shown here are obtained by 50 000 iterations excluding the initial 45 000 time steps. From the data shown above, we can find out that the time steps needed to reach the stable state in a three-route system are much longer than that in a two-route system, where it only needs 10 000 time steps to reach stable state.[22] So it brings about a lot of difficulties to our current work. The criterion for determining when a stable state is reached is that the value of average flux becomes stable. In other words, the value of average flux will not change with the increase of time steps. Figure 2 shows the dependence of average flux on expected arrival rate (V_p) at the entrance of the traffic system adopting four different feedback strategies. As to the route's processing capacity, we can see that in Fig. 2, there is a positive peak structure at the vicinity of $V_p \sim 0.82$ using VNFS and the flux is always the largest at each V_p value adopting VNFS. If you look carefully at this figure, you will find out that the slopes of these four curves decrease with the enhancement of V_p. This is caused by the fact that vehicles cannot enter the route immediately after

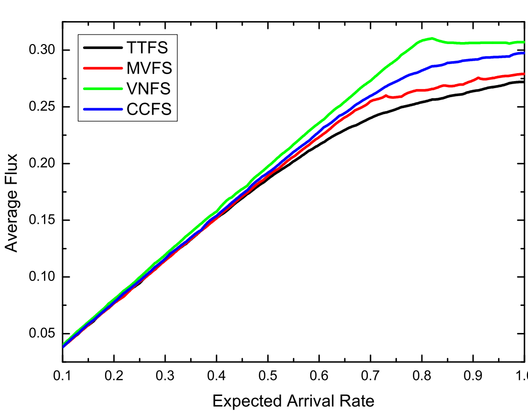

Fig. 2. Average flux vs expected arrival rate (V_p) at the entrance of the traffic system. The parameters are $L = 2000$, $p = 0.25$, and $S_{\text{dyn}} = 0.5$.

they arrive at the entrance when V_p increases to some value. The slope of the curve adopting VNFS is almost constant when $V_p \leq 0.82$ which means vehicles can always enter the route immediately after they arrive at the entrance until $V_p > 0.82$. Given that V_p at the entrance of traffic systems always changes at different times, we think VNFS is better than the other three information feedback strategies. Here, we are saying that V_p varies with time. For example, during time periods of 8:00–9:00 am and 5:00–6:00 pm, the value of V_p should be higher due to the fact that people go out to work and go back home. In this paper, we still set $V_p = 1$ as the former work.[20–22]

In contrast with VNFS, the fluxes of three routes adopting CCFS, MVFS, and TTFS show larger oscillation (see Fig. 3). This oscillation effect can be understood from the fact that the other three strategies cannot reflect the actual number of vehicles on each route. For example, there are 20 vehicles on route A where vehicles are distributed separately, 15 vehicles on route B where there exist two congestion

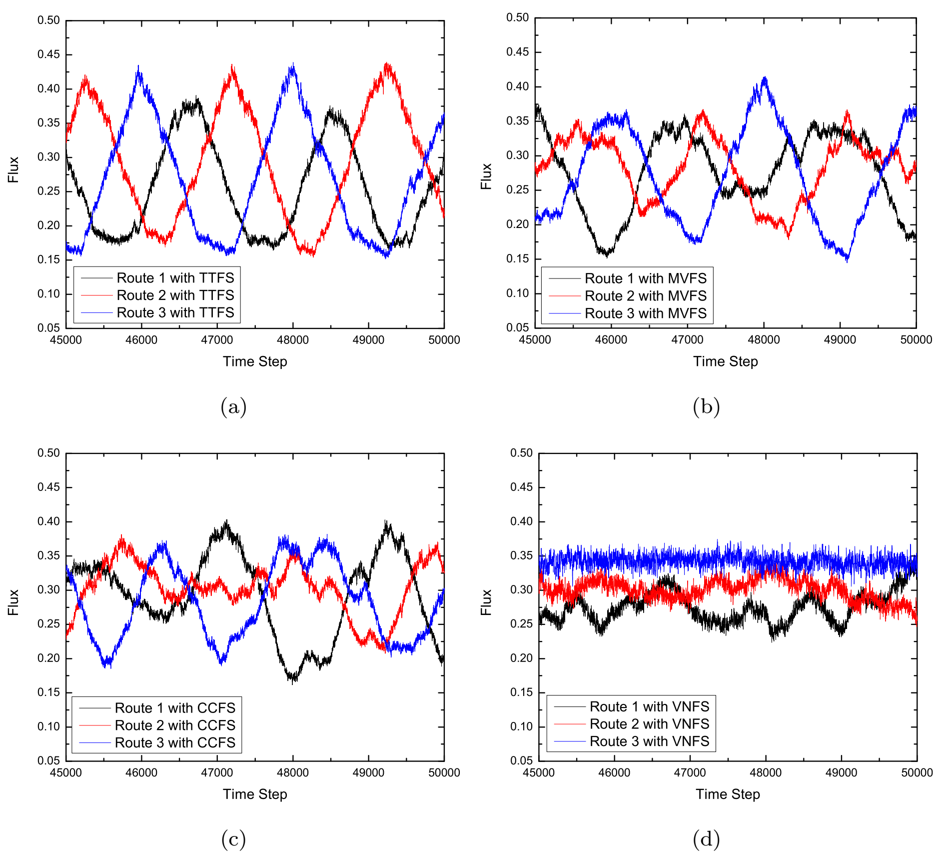

Fig. 3. (Color online) (a) Flux of each route with travel time feedback strategy. (b) Flux of each route with mean velocity feedback strategy. (c) Flux of each route with congestion coefficient feedback strategy. (d) Flux of each route with vehicle number feedback strategy. The parameters are $L = 2000$, $p = 0.25$, $S_{\text{dyn}} = 0.5$.

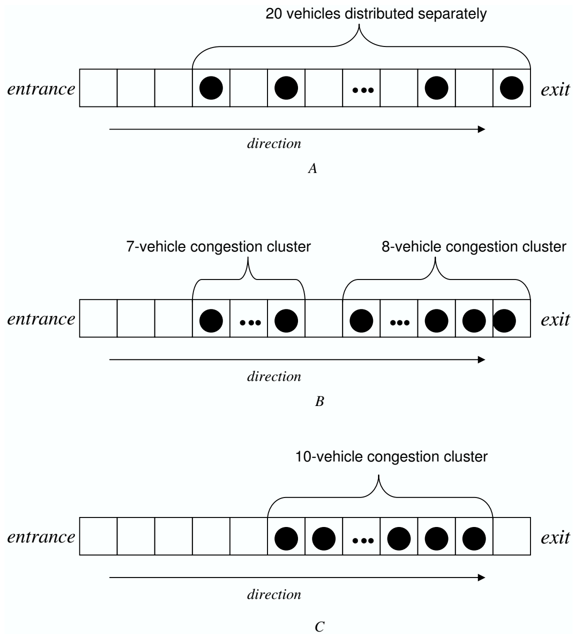

Fig. 4. Vehicle distribution on each route.

clusters and 10 vehicles on route C where vehicles are close to each other without a gap between any two of them (see Fig. 4). If we use CCFS, then the vehicles will enter route A instead of route B or route C, which will make more and more vehicles on route A. Furthermore, the oscillation may also be due to the information lag effect. For TTFS, the travel time reported by a driver at the end of three routes only represents the road condition in front of him, and perhaps the vehicles behind him have got into the jammed state. Unfortunately, this information will induce more vehicles to choose his route until a vehicle from the jammed cluster leaves the system. This effect apparently does harm to the system. For MVFS, we have mentioned that the NS model has a random brake scenario which causes the fragile stability of velocity, thus MVFS cannot completely reflect the real condition of routes. The other reason for the disadvantage of MVFS is that flux consists of two parts, mean velocity and vehicle density, but MVFS only grasps one part and lacks the other part of flux. Finally, the three-route system has only one exit, therefore, only one car can go out at each time step which may result in the traffic jam happening at the end of the routes. This may also cause the oscillation. Compared with CCFS, the performance adopting VNFS is remarkably improved, not only on the value but also the stability of the flux. Hence, with respect to the flux of the three-route system, VNFS is the best one.

In Fig. 5, vehicle number vs time step shows almost the same tendency as Fig. 3, and the routes' accommodating capacity is greatly enhanced with an increase in

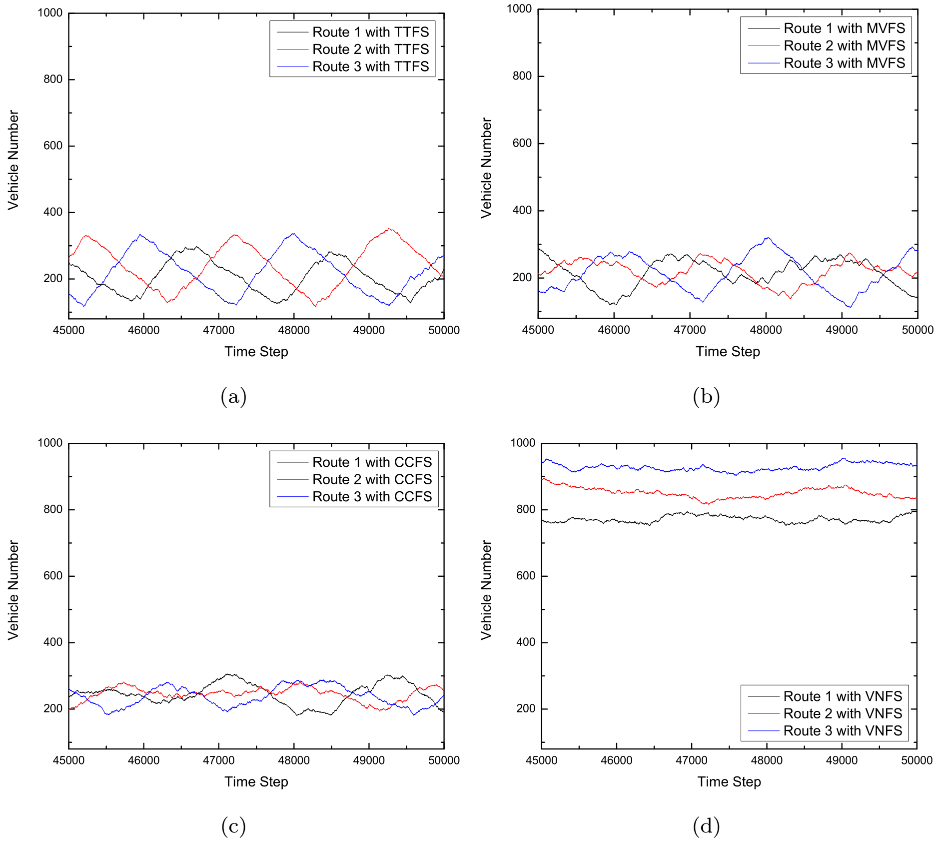

Fig. 5. (Color online) (a) Vehicle number of each route with travel time feedback strategy. (b) Vehicle number of each route with mean velocity feedback strategy. (c) Vehicle number of each route with congestion coefficient feedback strategy. (d) Vehicle number of each route with vehicle number feedback strategy. The parameters are set the same as in Fig. 3.

average vehicle number from 210 to 880, so perhaps the high fluxes of three routes with VNFS are mainly due to the increase of vehicle number.

In Fig. 6, speed vs time step shows that, although the speed is the stablest by using vehicle number feedback strategy, it is the lowest among the four different strategies. The reason is that the route's accommodating capacity is the best by using VNFS. As mentioned before, the traffic system has only one exit and only one vehicle can go out at each time step, thus more vehicles the lane owns, lower speeds the vehicles have. Fortunately, flux consists of two parts, mean velocity and vehicle density. Therefore, as long as the vehicle number (because the vehicle density is $\rho = N/L$, and the length L is fixed at 2000, so $\rho \propto$ vehicle number (N)) is large enough, the flux can also be the largest. The vehicle number and speed of Figs. 5(a)–5(c) and 6(a)–6(c) indicate that the traffic flow using TTFS, MVFS, and

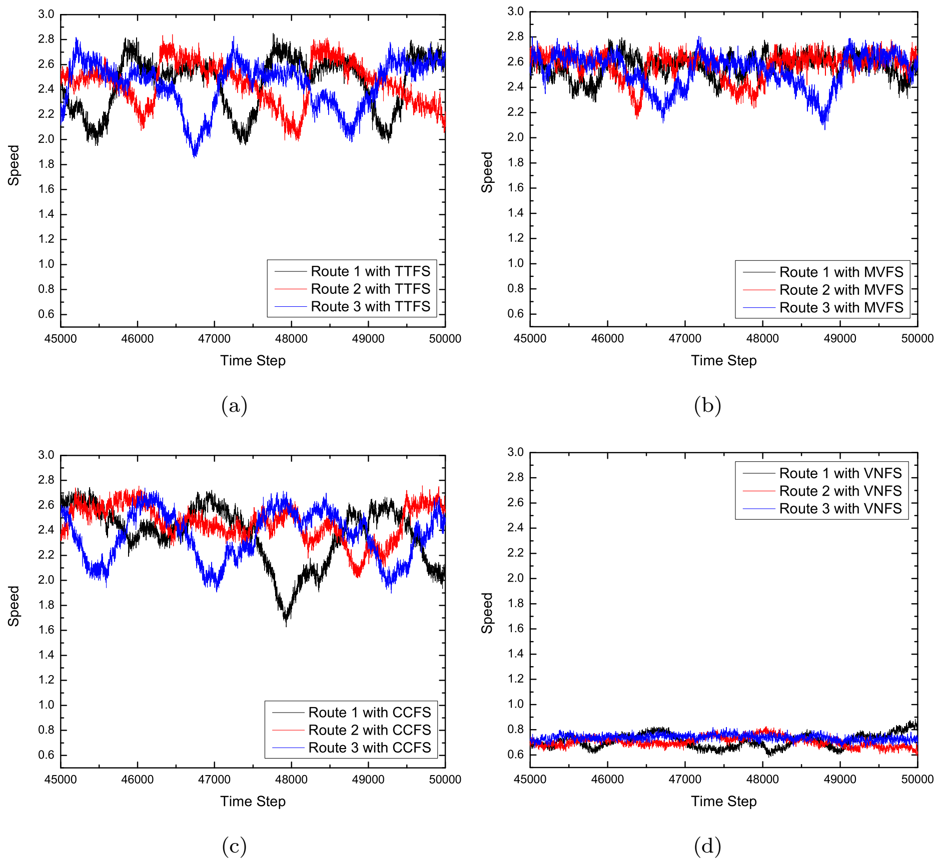

Fig. 6. (Color online) (a) Average speed of each route with travel time feedback strategy. (b) Average speed of each route with mean velocity feedback strategy. (c) Average speed of each route with congestion coefficient feedback strategy. (d) Average speed of each route with vehicle number feedback strategy. The parameters are set the same as in Fig. 3.

CCFS should be free flow. Also, from the stability of Figs. 5(d) and 6(d), we know that the traffic flow adopting VNFS should be synchronized flow.[27–30]

Someone may have doubts whether VNFS is really the best one due to the lowest speed shown in Fig. 6(d). In order to make the readers understand more easily, we assume that the road network under study includes not only the downstream corridor section displayed in Fig. 1, but also a section upstream of the entrance where vehicles wait to enter the corridor. Thus, we should take into account the total travel time (t_{tot}) that is the sum of driving time (t_{driving}) and waiting time (t_{waiting}) to evaluate the merits of these feedback strategies.

$$t_{\text{tot}} = t_{\text{driving}} + t_{\text{waiting}} \,. \tag{4}$$

Total travel time (t_{tot}): t_{tot} is the time period that vehicles spend on the whole traffic system that includes the upstream and downstream corridor sections. Flux

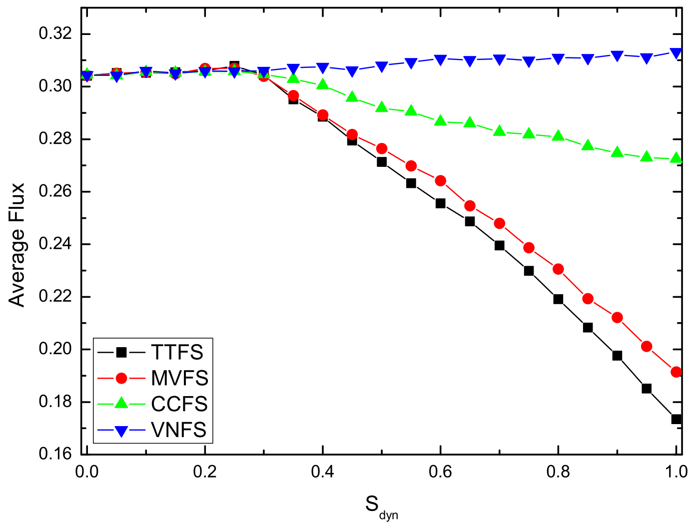

Fig. 7. (Color online) Average flux by performing four different feedback strategies vs S_{dyn}; L is fixed at 2000 and p is fixed at 0.25.

is a very good proxy to evaluate the total travel time, because it can be understood as the number of vehicles passing the exit at each time step. The more vehicles pass the exit during a fixed time period, the shorter time these vehicles spend on the traffic system. For example, there are totally N vehicles in the section upstream of the entrance, then the average flux of the traffic system is

$$F_{\text{avg}} = \frac{N}{t_{\text{tot (Nth vehicle)}}} \approx \frac{n}{t_{\text{tot (nth vehicle)}}}\,, \quad n \in [1, N]\,. \tag{5}$$

Here, one should be aware that the average flux value is stable; therefore, the approximation in Eq. (5) is valid. Thus, the travel time adopting VNFS is the shortest among these four feedback strategies for the fact that the flux adopting VNFS is the largest (see Fig. 7).

Driving time (t_{driving}): t_{driving} is the time period vehicles spend on the downstream route section displayed in Fig. 1. It is obvious that driving time of VNFS is the longest among these four feedback strategies since the speed v adopting VNFS is the lowest as shown in Fig. 6 (the length of the route L is fixed at 2000, thus $t_{\text{driving}} = L/v$).

Waiting time (t_{waiting}): t_{waiting} is the time period vehicles wait in the upstream route section. Since the total travel time (t_{tot}) is the sum of waiting time (t_{waiting}) and driving time (t_{driving}) as shown in Eq. (4), the waiting time adopting VNFS is the shortest on the basis of above analysis. Also, since the slope of the curve adopting VNFS always keeps constant until V_p increases to 0.82 (see Fig. 2), it indicates that the waiting time of VNFS is the shortest among these four feedback strategies. The slopes of the curves in Fig. 2 becoming lower means more vehicles cannot enter the route immediately after they arrive at the entrance.

Figure 7 shows that the average flux fluctuates feebly with a persisting increase of dynamic travelers by using four different strategies. As to the route's processing capacity, the vehicle number feedback strategy is proved to be the best one because the flux is always the largest at each $S_{\rm dyn}$ value and even enhances with a persisting increase of dynamic travelers. A question here for some readers is why the average fluxes in Fig. 7 using four different strategies are smaller than that shown in the former work[24] where the traffic system has only two routes. The reason is that the three-route system in this paper still permits at most one vehicle to enter the entrance at each time step. Hence, the more routes the traffic system owns, the lower average flux it has. Furthermore, we compute the average flux in a two-route system adopting VNFS ($S_{\rm dyn} = 1$) and the value is about 0.385, so the total flux of the system should be $\sum_{j=1}^{2} F_j = 0.385 \times 2 = 0.77$. The average flux in a three-route system adopting VNFS ($S_{\rm dyn} = 1$) is about 0.31 (see Fig. 7), thus the total flux of the system should be $\sum_{j=1}^{3} F_j = 0.31 \times 3 = 0.93$. Hence, if the route number in a multi-route system is more than three, the total flux will further increase until the system allows vehicles to enter the route immediately after they arrive at the entrance.

4. Conclusion

We obtain the simulation results of applying four different feedback strategies, i.e. TTFS, MVFS, CCFS, and VNFS in a three-route scenario all with respect to flux, number of vehicles, speed, average flux vs expected arrival rate (V_p) and average flux vs $S_{\rm dyn}$. The results indicate that VNFS has more advantages than the other three strategies in the three-route system which has only one entrance and one exit. We also find that it will take much more time to reach the stable state in the three-route system than that in the two-route system. In contrast with the other three feedback strategies, VNFS can significantly improve the road condition, including increasing vehicle number and flux, reducing oscillation and total travel time, and that average flux enhances with the increase of $S_{\rm dyn}$. The numerical simulation results demonstrate that the expected arrival rate (V_p) plays a very important role in improving the road condition. Therefore, if we can find out an advanced information feedback strategy which can control V_p at the entrance of the traffic system, the flux of each route will be further improved.

Due to the rapid development of modern scientific technology, it is easy to realize VNFS in reality, if only some pressure transducers are installed at the entrance and exit of each route, respectively. Furthermore, doing so will cost less than CCFS because pressure transducers are always cheaper than GPS. Also, we only need to install six pressure transducers in a three-route system, instead of installing a navigation system (GPS) in each vehicle as CCFS.[22] Taking into account the reasonable cost and more accurate description of road condition, we think this new feedback strategy will be applicable.

Acknowledgments

C.-F. Dong would like to thank Dr. N. Liu at the University of Chicago and the referees for some helpful comments while we were preparing the manuscript.

This work has been partially supported by Georgia Institute of Technology, the National Basic Research Program of China (973 Program No. 2006CB705500), the National Natural Science Foundation of China (Grant Nos. 10975126 and 10635040), and the Specialized Research Fund for the Doctoral Program of Higher Education of China (Grant No. 20093402110032).

References

1. D. Chowdhury, L. Santen and A. Schadschneider, *Phys. Rep.* **329**, 199 (2000).
2. D. Helbing, *Rev. Mod. Phys.* **73**, 1067 (2001).
3. T. Nagatani, *Rep. Prog. Phys.* **65**, 1331 (2002).
4. R. W. Rothery, in *Transportation Research Board Special Report*, Vol. 165, eds. N. Gartner, C. J. Messner and A. J. Rathi (Transportation Research Board, Washington, D. C., 1992), Chap. 4.
5. I. Prigogine and F. C. Andrews, *Oper. Res.* **8**, 789 (1960).
6. S. L. Paveri-Fontana, *Transp. Res.* **9**, 225 (1975).
7. H. Lehmann, *Phys. Rev. E* **54**, 6058 (1996).
8. C. Wagner, C. Hoffmann, R. Sollacher, J. Wagenhuber and B. Schürmann, *Phys. Rev. E* **54**, 5073 (1996).
9. D. Helbing, *Phys. Rev. E* **53**, 2366 (1996).
10. D. Helbing, *Phys. Rev. E* **57**, 6176 (1997).
11. D. Helbing and M. Treiber, *Phys. Rev. Lett.* **81**, 3042 (1998).
12. K. Nagel and M. Schreckenberg, *J. Phys. I* **2**, 2221 (1992).
13. O. Biham, A. A. Middleton and D. Levine, *Phys. Rev. A* **46**, R6124 (1992).
14. Y. Yokoya, *Phys. Rev. E* **69**, 016121 (2004).
15. T. L. Friesz, J. Luque, R. L. Tobin and B.-W. Wie, *Oper. Res.* **37**, 893 (1989).
16. M. Ben-Akiva, A. de Palma and I. Kaysi, *Transp. Res., Part A* **25A**, 251 (1991).
17. H. S. Mahmassani and R. Jayakrishnan, *Transp. Res., Part A* **25A**, 293 (1991).
18. R. Arnott, A. de Palma and R. Lindsey, *Transp. Res., Part A* **25A**, 309 (1991).
19. P. Kachroo and K. Özbay, *Transp. Res. Rec.* **1556**, 137 (1996).
20. J. Wahle, A. L. C. Bazzan, F. Klügl and M. Schreckenberg, *Physica A* **287**, 669 (2000).
21. K. Lee, P.-M. Hui, B.-H. Wang and N. F. Johnson, *J. Phys. Soc. Jpn.* **70**, 3507 (2001).
22. W.-X. Wang, B.-H. Wang, W.-C. Zheng, C.-Y. Yin and T. Zhou, *Phys. Rev. E* **72**, 066702 (2005).
23. B.-H. Wang, D. Mao and P.-M. Hui, *Proc. Second International Symposium on Complexity Science*, Shanghai, August 6–7, 2002, p. 204.
24. C.-F. Dong, X. Ma and B.-H. Wang, *Phys. Lett. A* **374**, 1326 (2010).
25. N. Moussa, *Int. J. Mod. Phys. C* **15**, 29 (2004).
26. X.-G. Li, Z.-Y. Gao, B. Jia and X.-M. Zhao, *Int. J. Mod. Phys. C* **20**, 501 (2009).
27. B. S. Kerner and H. Rehborn, *Phys. Rev. Lett.* **79**, 4030 (1997); *Phys. Rev. E* **53**, R1297 (1996); *Phys. Rev. E* **53**, R4275 (1996).
28. B. S. Kerner, *Phys. Rev. Lett.* **81**, 3797 (1998); *Phys. Rev. E* **65**, 046138 (2002).
29. R. Jiang and Q.-S. Wu, *Eur. Phys. J. B* **46**, 581 (2005).
30. D. Helbing, A. Hennecke and M. Treiber, *Phys. Rev. Lett.* **82**, 4360 (1999).