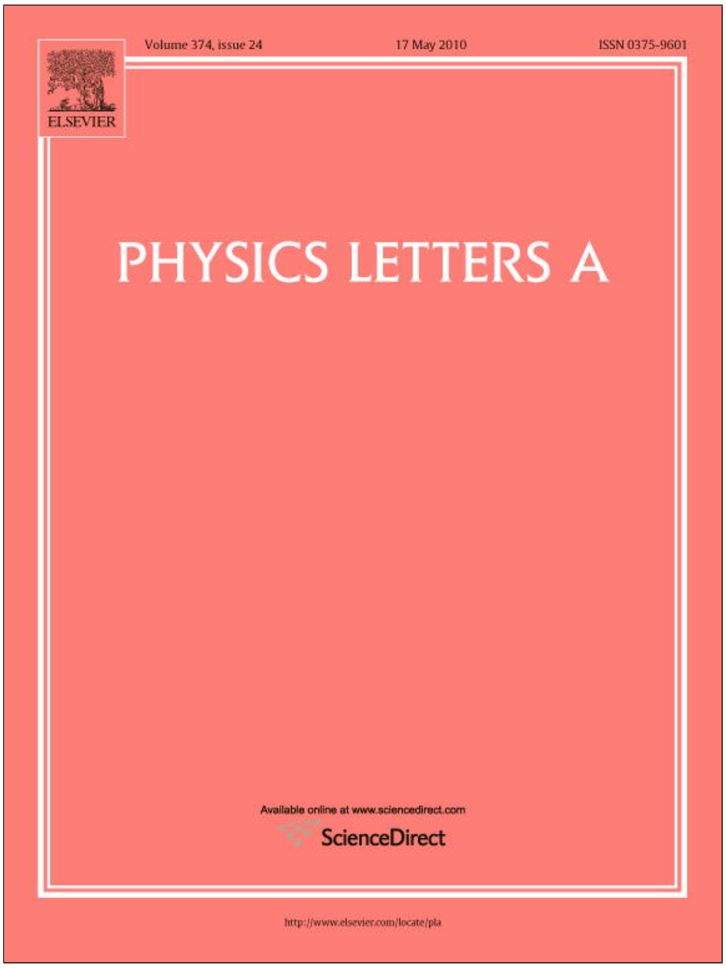
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Corresponding Angle Feedback in an innovative weighted transportation system

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ABSTRACT

The optimal information feedback has a significant effect on many socioeconomic systems like stock market and traffic systems aiming to make full use of resources. In this Letter, we study dynamics of traffic flow with real-time information. The influence of a feedback strategy named Corresponding Angle Feedback Strategy (CAFS) is introduced, based on a two-route scenario in which dynamic information can be generated and displayed on the board to guide road users to make a choice. Our model incorporates the effects of adaptability into the cellular automaton models of traffic flow and simulation results adopting this optimal information feedback strategy have demonstrated high efficiency in controlling spatial distribution of traffic patterns compared with the other three information feedback strategies, i.e., vehicle number and flux.

1. Introduction

Physics, other sciences and technologies meet at the frontier area of interdisciplinary research. The concepts and techniques of physics are being applied to such complex systems as transportation systems [1–5]. A lot of theories have been proposed such as car-following theory [6–8], kinetic theory [9–14] and particle-hopping theory [15–18]. These theories have the advantages of alleviating the traffic congestion and enhancing the capacity of existing infrastructure. Although dynamics of traffic flow with real-time traffic information have been extensively investigated [19–24], finding out a more efficient feedback strategy is still an overall task. Recently, some information feedbacks have been put forward to investigate the two-route scenario with the same length. Wahle et al. [25,26] first investigated the two-route scenario with travel time feedback strategy (TTFS). Subsequently, Lee et al. [27] studied the effect of a different type of information feedback (MVFS), i.e. instantaneous average velocity. Then Wang et al. [28] proposed a third type of information feedback (CCFS), i.e. instantaneous congestion coefficient which is defined as $C = \sum_{i=1}^{p} n_i^2$. Here, n_i stands for vehicle number of the ith congestion cluster in which cars are close to each other without a gap between any two of them; p is the number of congestion clusters. Furthermore, Dong et al. [29] put forward another type of information feedback (WCCFS), i.e. instantaneous weighted congestion coefficient which is defined as $C_W = \sum_{i=1}^{p} F(n_m) n_i^2 = \sum_{i=1}^{p} (k \times \frac{n_m}{2000} + 2.0) \times n_i^2$. Here, the definition of n_i is the same as above, and $F(n_m)$ is the weight function. It has been proved that TTFS is the worst one which brings a lag effect to make it impossible to provide the road users with the real situation of each route [27]; CCFS is more efficient than MVFS because the random brake mechanism of the Nagel–Schreckenberg (NS) model [15] brings fragile stability of velocity [28]; and WCCFS is more efficient than CCFS for the reason that CCFS doesn't take the weights of the route into consideration [29]. However, WCCFS is still not the best one due to the fact that the weight function $F(n_m)$ doesn't contain any information related to the length of the congestion cluster and some other reasons will be discussed delicately in this Letter. In order to provide road users with better guidance, a strategy named Corresponding Angle Feedback Strategy (CAFS) is presented. We report the simulation results adopting four different feedback strategies MVFS, CCFS, WCCFS and CAFS in a two-route scenario with a single route following the NS mechanism.

The Letter is arranged as follows: In Section 2, the NS model and a two-route scenario are briefly introduced, together with four feedback strategies of MVFS, CCFS, WCCFS and CAFS all depicted in more details. In Section 3, some simulation results will be presented and discussed based on the comparison of four different feedback strategies. In the last section, we will make some conclusions.

2. The model and feedback strategies

2.1. NS mechanism

The Nagel–Schreckenberg (NS) model is so far the most popular and simplest cellular automaton model in analyzing the traf-

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fic flow [3–5,15,30], where the one-dimension CA with periodic boundary conditions is used to investigate highway and urban traffic. This model can reproduce the basic features of real traffic like stop-and-go wave, phantom jams, and the phase transition on a fundamental diagram. In this section, the NS mechanism will be briefly introduced as a base of analysis.

The road is subdivided into cells with a length of $\Delta x = 7.5$ m. Let N be the total number of vehicles on a single route of length L, then the vehicle density is $\rho = N/L$. $g_n(t)$ is defined to be the number of empty sites in front of the nth vehicle at time t, and $v_n(t)$ to be the speed of the nth vehicle, i.e., the number of sites that the nth vehicle moves during the time step t. In the NS model, the maximum speed is fixed to be $v_{\max} = M$. In the present Letter, we set $M = 3$ for simplicity.

The NS mechanism can be decomposed to the following four rules (parallel dynamics):

Rule 1. Acceleration: $v_i \leftarrow \min(v_i + 1, M)$;
Rule 2. Deceleration: $v'_i \leftarrow \min(v_i, g_i)$;
Rule 3. Random brake: with a certain brake probability p do $v''_i \leftarrow \max(v'_i - 1, 0)$; and
Rule 4. Movement: $x_i \leftarrow x_i + v''_i$.

The fundamental diagram characterizes the basic properties of the NS model which has two regimes called "free-flow" phase and "jammed" phase. The critical density, basically depending on the random brake probability p, divides the fundamental diagram to these two phases.

2.2. Two-route scenario

Wahle et al. [25] first investigated the two-route model in which road users choose one of the two routes according to the real-time information feedback. In a two-route scenario, it is supposed that there are two routes A and B of the same length L. At each time step, a new vehicle is generated at the entrance of two routes and will choose one route. If a vehicle enters one of two routes, the motion of it will follow the dynamics of the NS model. As a remark, if a new vehicle is unable to enter the desired route, it will be deleted. And a vehicle will also be removed after it reaches the end point.

Additionally, two types of vehicles are introduced: dynamic and static vehicles. If a driver is a so-called dynamic one, he will make a choice on the basis of the information feedback [25], while a static one just enters a route at random ignoring any advice. The density of dynamic and static travelers are S_{dyn} and $1 - S_{dyn}$, respectively.

The simulations are performed by the following steps: first, we set the routes and board empty; second, after the vehicles enter the routes, according to four different feedback strategies, information will be generated, transmitted, and displayed on the board at each time step. Finally, the dynamic road users will choose the route with better condition according to the dynamic information at the entrance of two routes.

2.3. Related definitions

The road conditions can be characterized by fluxes of two routes, and flux is defined as follows:

$$F = V_{mean}\rho = V_{mean}\frac{N}{L} \tag{2.1}$$

where V_{mean} represents the mean velocity of all the vehicles on one of the roads, N denotes the vehicle number on each road, and L is the length of two routes. Then we describe four different feedback strategies, respectively.

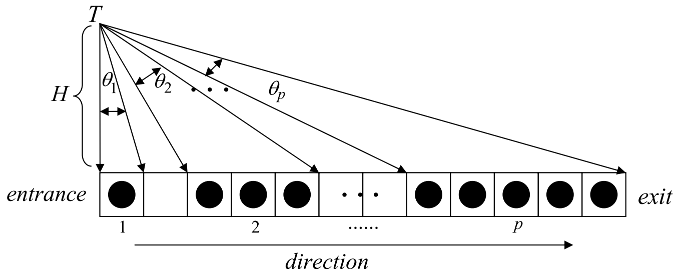

Fig. 1. Angles corresponding to each congestion cluster on the lane.

MVFS: Every time step, each vehicle on the routes transmits its velocity to the traffic control center which will deal with the information and display the mean velocity of vehicles on each route on the board. Road users at the entrance will choose one road with larger mean velocity.

CCFS: Every time step, each vehicle transmits its signal to satellite, then the navigation system (GPS) will handle that information and calculate the position of each vehicle which will be transmitted to the traffic control center. The work of the traffic control center is to compute the congestion coefficient of each road and display it on the board. Road users at the entrance will choose one road with smaller congestion coefficient.

The congestion coefficient is defined as

$$C = \sum_{i=1}^{p} n_i^w. \tag{2.2}$$

Here, n_i stands for vehicle number of the ith congestion cluster in which cars are close to each other without a gap between any two of them. Every cluster is evaluated by a weight w, here $w = 2$ [28].

WCCFS: Every time step, each vehicle transmits its signal to satellite, then the navigation system (GPS) will handle that information and calculate the position of each vehicle which will be transmitted to the traffic control center. The work of the traffic control center is to compute the congestion coefficient of each road with a reasonable weighted function and display it on the board. Road users at the entrance will choose one road with smaller weighted congestion coefficient.

The weighted congestion coefficient is defined as [29]

$$C_w = \sum_{i=1}^{p} F(n_m)n_i^w = \sum_{i=1}^{p}\left(k \times \frac{n_m}{2000} + 2.0\right) \times n_i^w \tag{2.3}$$

where n_i stands for vehicle number of the ith congestion cluster in which cars are close to each other without a gap between any two of them. n_m stands for the position of the ith congestion cluster, for example, if the vehicle number of the ith congestion cluster is odd, the position of the median of congestion cluster stands for the position of it, and if the vehicle number of the ith congestion cluster is even, then we use the result of median rounding $\lfloor n_m \rfloor$ to represent the position of the congestion cluster. And $F(x)$ stands for the weighted function of each route. Here $w = 2$ as CCFS [28].

CAFS: Every time step, the traffic control center will receive data from the navigation system (GPS) like WCCFS. The work of the traffic control center is to compute the corresponding angle of each congestion cluster (see Fig. 1) on the lane, sum square of each corresponding angle up and display it on the board. Road users at the entrance will choose one road with smaller corresponding angle coefficient.

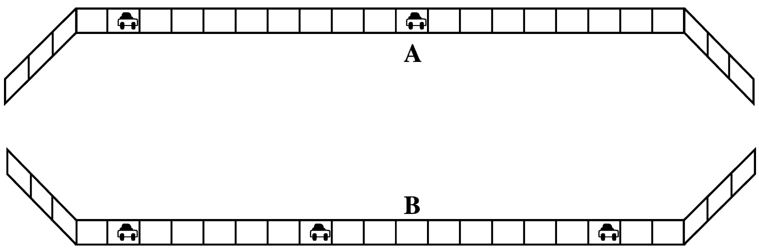

Fig. 2. The one entrance and one exit two-route traffic system.

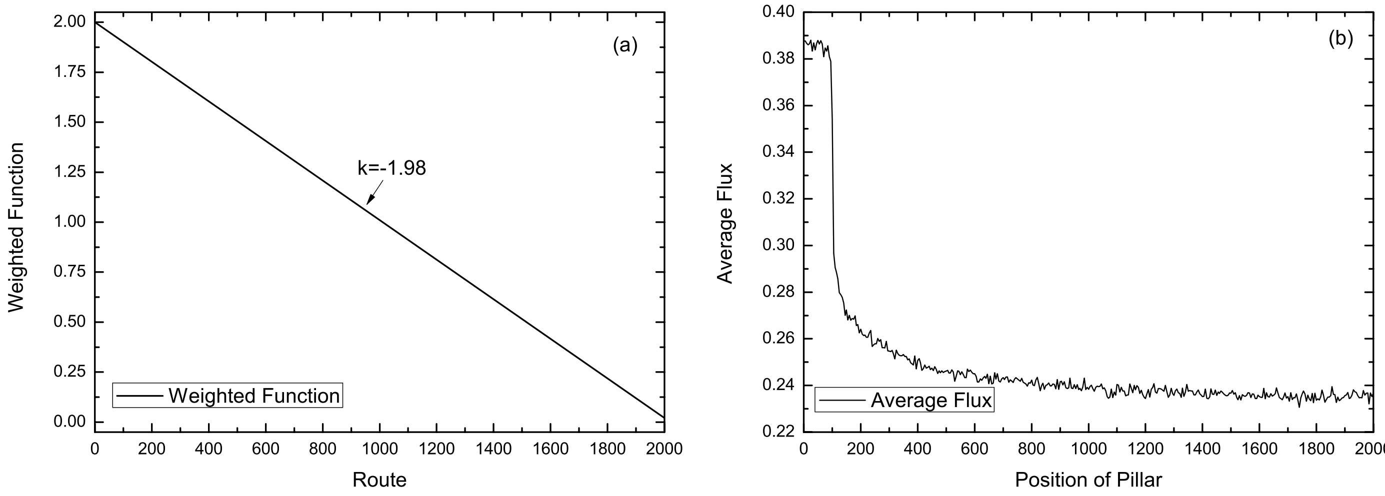

Fig. 3. (a) Weight value of each site of the route [29]. (b) Average flux vs position of pillar (point T). The parameters are $L = 2000$, $p = 0.25$, $S_{dyn} = 1.0$ and vertical distance (H) is fixed to be 100.

The corresponding angle coefficient is defined as

$$C_\theta = \sum_{i=1}^{p} \theta_i^2 = \sum_{i=1}^{p} \left(\arctan\left(\frac{n_i^{first}}{H}\right) - \arctan\left(\frac{n_i^{first} - n_i^{last}}{H}\right)\right)^2 \tag{2.4}$$

where n_i^{first} and n_i^{last} stand for the position of the first and last vehicle in the ith congestion cluster, in which vehicles are close to each other without a gap between any two of them, respectively. θ_i stands for the weight (corresponding angle) of the ith congestion cluster. H denotes the vertical distance from the point T to the lane, and in this Letter, we set $H = 100$.

In this Letter, the two-route system has only one entrance and one exit as shown in Fig. 2. The rules at the exit of the two-route system are as follows:

(a) At the end of two routes, the car that is nearer to the exit goes first.

(b) If the cars at the end of two routes have the same distance to the exit, faster one drives, first it goes out.

(c) If the cars at the end of two routes have the same speed and distance to the exit, the car in the route which owns more cars drives out first.

(d) If the rules (a), (b) and (c) are satisfied at the same time, then the cars go out randomly.

In the following section, performance by using four different feedback strategies will be shown and discussed in more details.

3. Simulation results

All simulation results shown here are obtained by 15000 iterations excluding the initial 5000 time steps. Fig. 3(a) shows the weight value of the entrance is much larger than that of the exit when adopting WCCFS [29]. Thus, the point T located above the entrance of the route (see Fig. 1) is reasonable. This makes the weight of the entrance is the largest. Furthermore, the corresponding angle of each congestion cluster can reflect not only the weight of the route but also the length of the congestion cluster. So the weight value is more reasonable than before. Fig. 3(b) shows the dependence of average flux on position of the pillar (point T) by using the new strategy. As to the routes' processing capacity, we can see that the position of the pillar will directly affect the average flux. The average flux is much larger when point T locates at the entrance of the route while pretty lower when point T locates at the end of the lane. Therefore, the result is in accord with Fig. 3(a). Also, this can be understood as shown in Fig. 4. In Fig. 4, the congestion cluster on route A locates at the entrance of lane and the congestion cluster on route B locates at the end of the lane. You can see from the figure very clearly that C_θ of route A is larger than that of route B, so the road user should enter route B instead of route A. If point T locate at the end of the route, C_θ of route A will smaller than that of route B, then the vehicle will select route A to enter, which will make the cluster larger or the vehicle even cannot enter the route.

In contrast with CAFS, the fluxes of two routes adopting CCFS, MVFS and WCCFS show larger oscillation (see Fig. 5). This oscillation effect can be understood for several reasons. On one hand, CCFS and MVFS cannot reflect the weights of different parts of the lane. One the other hand, though WCCFS can reflect the route weights, the weighted coefficient ($F(n_m)$) of n_i^2 has no relationship to the length of congestion cluster, because we use the median of congestion cluster to represent its position when adopting WCCFS. Also, the two-route system only has one exit, therefore, only one car can go out at each time step. This may result in the traffic jam to happen at the end of the routes. However, the new strategy can make the value of congestion coefficient at the end of the routes smaller than before which is equivalent to alleviate the negative effects of congestion caused by the traffic jam. Meanwhile, the new strategy also take the length of the congestion cluster into account, which can give the road user with better guidance. For example, if there exit congestion clusters at the end of both routes, the road user will choose the route with shorter cluster length, because there is a positive correlation between the value of C_θ and the length of the cluster under this situation. Hence, the new strategy may improve the road situation. Compared to WC-CFS, the performance adopting CAFS is remarkably improved, not only on the value but also the stability of the flux. Therefore, as to the flux of the two-route system, CAFS is the best one.

In Fig. 6, vehicle number versus time step shows almost the same tendency as Fig. 5, and that the routes' accommodating capacity is greatly enhanced with an increase in average vehicle number from 310 to 785, so perhaps the high fluxes of two routes with CAFS are mainly due to the increase of vehicle number.

In Fig. 7, speed versus time step shows that although the speed is the stablest by using the new strategy, it is the lowest among the four different strategies. The reason is that the routes' accommodating capacity is the best by using the new strategy. And as mentioned before, the road has only one exit and only one car can go out at each time step. Therefore, more cars the lane owns, lower speeds the vehicles have. Fortunately, flux consists of two parts, mean velocity and vehicle density. Hence, as long as the ve-

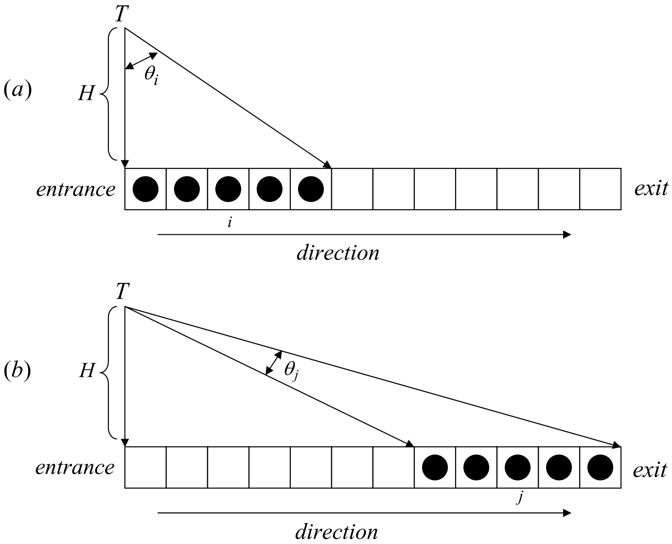

Fig. 4. The location and corresponding angle of vehicle congestion cluster on route A and route B.

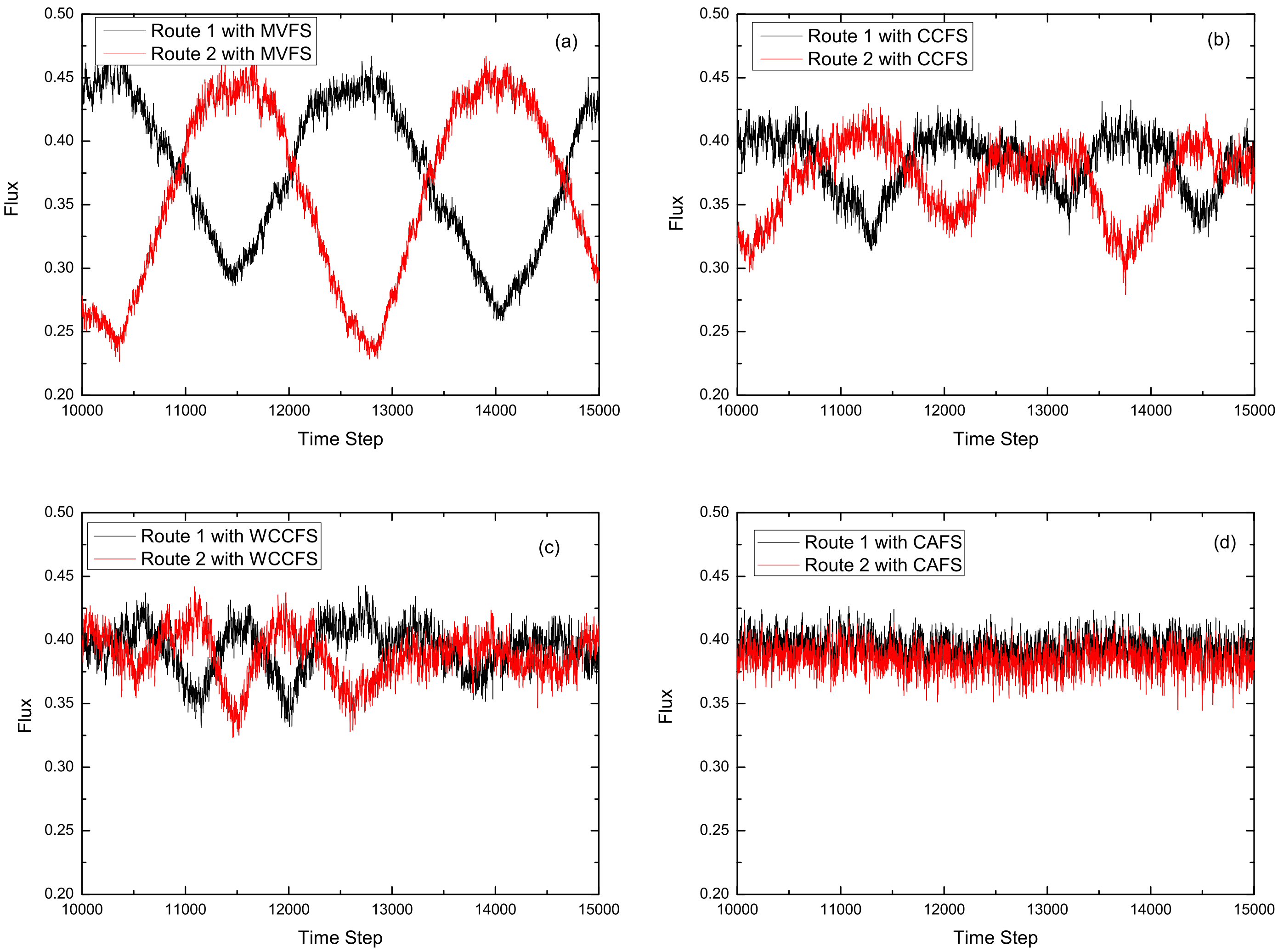

Fig. 5. (Color online.) (a) Flux of each route with mean velocity feedback strategy. (b) Flux of each route with congestion coefficient feedback strategy. (c) Flux of each route with weighted congestion coefficient feedback strategy. (d) Flux of each route with corresponding angle feedback strategy. The parameters are $L = 2000$, $p = 0.25$, $S_{dyn} = 0.5$, and vertical distance (H) is fixed to be 100.

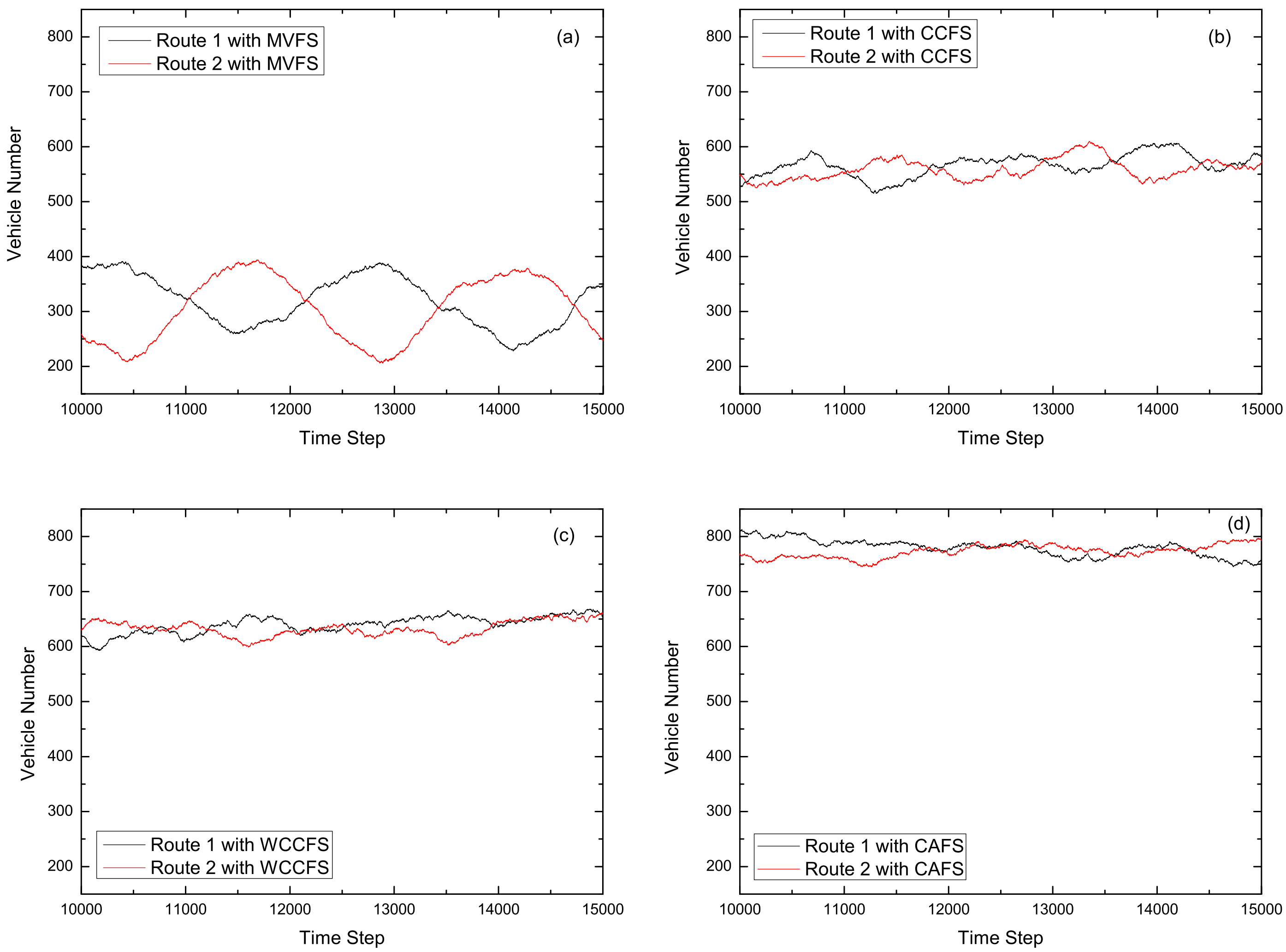

Fig. 6. (Color online.) (a) Vehicle number of each route with mean velocity feedback strategy. (b) Vehicle number of each route with congestion coefficient feedback strategy. (c) Vehicle number of each route with weighted congestion coefficient feedback strategy. (d) Vehicle number of each route with corresponding angle feedback strategy. The parameters are set the same as in Fig. 5.

hicle number (because the vehicle density is $\rho = N/L$, and the L is fixed to be 2000, so $\rho \propto$ vehicle number (N)) is large enough, the flux can also be the largest.

Fig. 8 shows that the average flux fluctuates feebly with a persisting increase of dynamic travelers by using four different strategies. As to the routes' processing capacity, the new strategy is proved to be the best one because the flux is always the largest at each S_{dyn} value and keeps the two routes' fluxes in balance.

4. Conclusion

We obtain the simulation results of applying four different feedback strategies, i.e., MVFS, CCFS, WCCFS and CAFS in a two-route scenario all with respect to flux, number of cars, speed, and average flux versus S_{dyn}. We also show the results about average flux versus position of pillar (point T) adopting the new information feedback strategy. These results indicate that CAFS has more advantages than the other three strategies in the two-route system with only one entrance and one exit. The highlight of this Letter is that it brings forward a new quantity namely corresponding angle (θ) to radically improve the road conditions. In contrast with other three feedback strategies, CAFS can significantly improve the road conditions, including increasing vehicle number and flux, reducing oscillation, and that average flux enhances with the increase of S_{dyn}. And it can be understood because the new strategy can reflect the weights of different parts of the route and take the length of each congestion cluster into consideration at the same time. Furthermore, the new strategy can alleviate the negative effects of congestion caused by the traffic jam at the end of the route. The numerical simulations demonstrate that the position of point T plays a very important role in improving the road situation.

Due to the rapid development of modern scientific technology, it is not difficult to realize CAFS in reality. If only a navigation system (GPS) is installed in each vehicle, thus the position information of vehicles will be known. Then CAFS can come true through computational simulation by calculate the corresponding angle of each congestion cluster on the lane and sum square of these angles up. Also, it will cost no more than WCCFS because the computers using to compute the weighted congestion coefficient can also calculate the corresponding angle. Taking into account the reasonable cost and more accurate description of road condition, we think this new feedback strategy shall be applicable.

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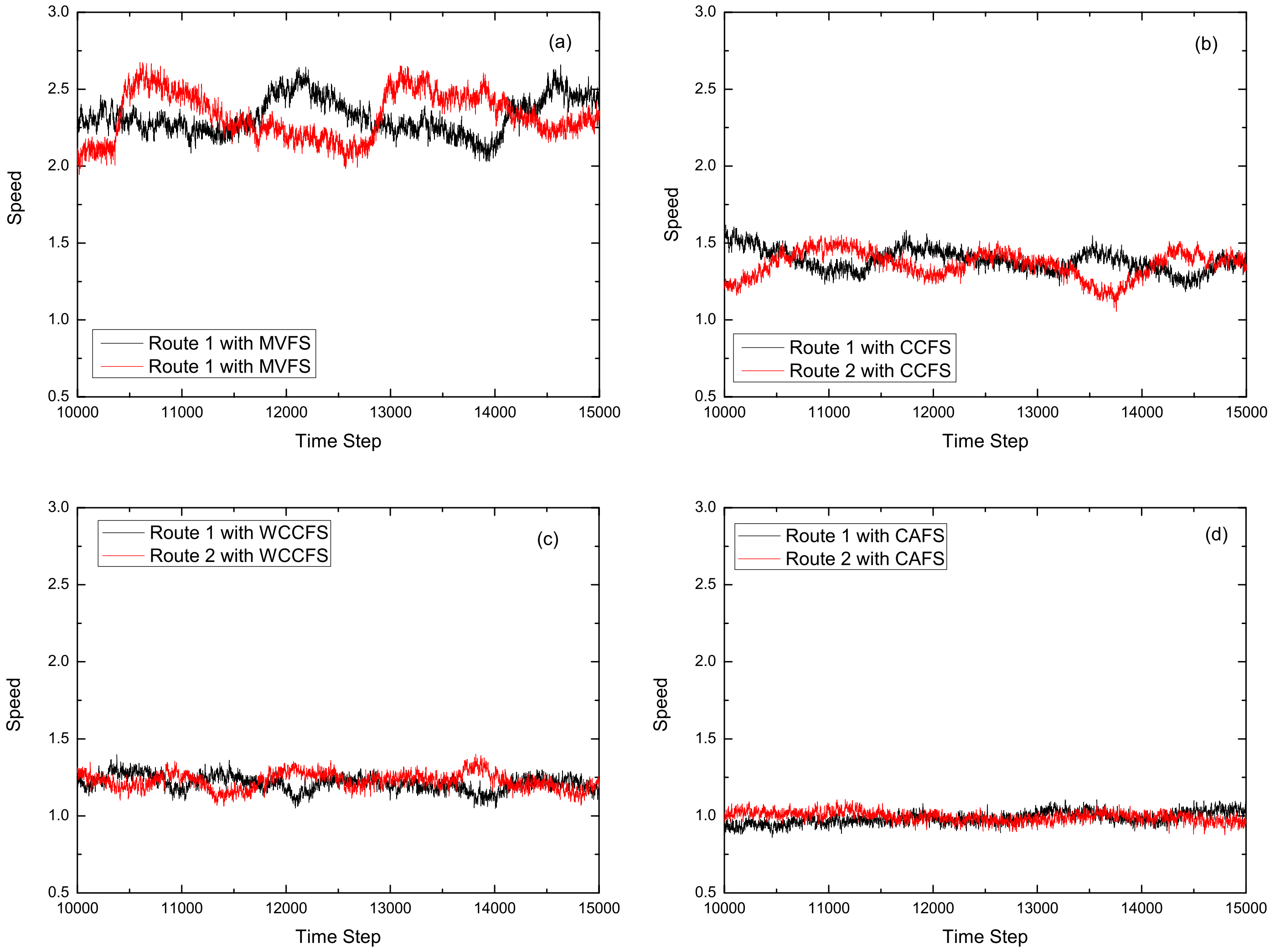

Fig. 7. (Color online.) (a) Average speed of each route with mean velocity feedback strategy. (b) Average speed of each route with congestion coefficient feedback strategy. (c) Average speed of each route with weighted congestion coefficient feedback strategy. (d) Average speed of each route with corresponding angle feedback strategy. The parameters are set the same as in Fig. 5.

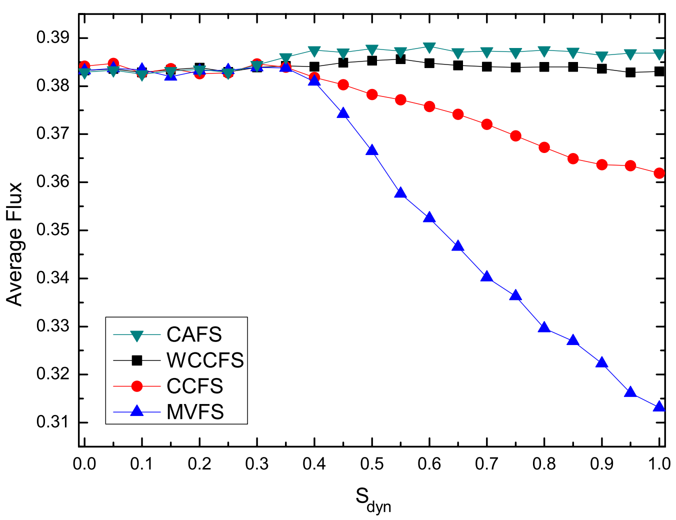

Fig. 8. (Color online.) Average flux by performing different strategy vs S_{dyn}; L is fixed to be 2000, p is fixed to be 0.25 and vertical distance (H) is fixed to be 100.

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