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PIECEWISE FUNCTION FEEDBACK STRATEGY IN INTELLIGENT TRAFFIC SYSTEMS WITH A SPEED LIMIT BOTTLENECK

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The road capacity can be greatly improved if an appropriate and effective information feedback strategy is adopted in the traffic system. In this paper, a strategy called piecewise function feedback strategy (PFFS) is introduced and applied into an asymmetrical two-route scenario with a speed limit bottleneck in which the dynamic information can be generated and displayed on the information board to guide road users to make a choice. Meanwhile, the velocity-dependent randomization (VDR) mechanism is adopted which can better reflect the dynamic behavior of vehicles in the system than NS mechanism. Simulation results adopting PFFS have demonstrated high efficiency in controlling spatial distribution of traffic patterns compared with the previous strategies.

1. Introduction

Traffic flow phenomena have attracted great interest of physicists since the early 1990s. Many models and analysis have been carried out from the viewpoint of statistical physics by various groups in recent years in order to explain empirical findings.[1−4] The effect of real traffic conditions has also been investigated.[5,6] At the same time, advanced traveler information systems provide current or even predictive information about the traffic flow to the road users to alleviate traffic congestion and enhance the capacity of the existing infrastructure.[7−9] Although the dynamics of traffic flow with real-time traffic information have been extensively investigated,[10,11] finding a more efficient feedback strategy is a forever-important task. In recent years, some information feedback strategies have been put forward to investigate the two-route scenario with the same length, such as the travel time feedback strategy (TTFS),[12] mean velocity feedback strategy (MVFS)[13] and congestion coefficient feedback strategy (CCFS).[14] It has been proved that MVFS is more efficient than TTFS which brings a lag effect, making it impossible to provide the road users with real situation of each route[13] and CCFS is more efficient than MVFS because of the fact that the random brake mechanism of the Nagel−Schreckenberg (NS) model[15] brings fragile stability of velocity.[14] Recently, Dong *et al.* proposed one straightforward and concise model called weighted congestion coefficient feedback strategy (WCCFS).[16] Compared with the other information feedback strategies, the new one can reflect the route weight and prevent the congestion cluster from further expanding, which in turn alleviates the jammed state of the system. WCCFS, however, is still not the best one. In the real world, many two-route systems are asymmetric. For example, two-route lengths are different or there is only one bottleneck in one of the routes. In these cases, it is not appropriate to adopt MVFS, CCFS or WCCFS as the feedback strategy.

In our daily life, road conditions are usually changed by some accidents such as traffic accident, bad weather and the policy of speed limit, etc. Based on this idea, we put forward an information feedback strategy called piecewise function feedback strategy (PFFS). This strategy can reflect the weight of the same congestion clusters in different parts of the route even if in the bottleneck. Here, we adopt the velocity-dependent randomization (VDR) mechanism to avoid the influence of velocity instability which is caused by the random decreasing probability in NS mechanism.

2. Related Definitions

2.1. *The VDR mechanism*

Barlovic *et al.* suggested a cellular automata model called VDR model.[17] In this model, due to the variables of discrete time and space, the cars update their positions and velocities according to the following rules:

(1) Acceleration: $v_i(t) \to v_i(t+1/3) = \min\{v_i(t)+1, v_{\max}\}$,
(2) Deceleration: $v_i(t+1/3) \to v_i(t+2/3) = \min\{v_i(t+1/3), d_i(t)\}$,
(3) Randomization with probability $p_n(t+1)$:

$$p_n(t+1) = \begin{cases} p_0, & v_i(t) = 0; \\ p_d, & v_i(t) > 0. \end{cases}$$

$$v_i\left(t+\frac{2}{3}\right) \to v_i(t+1) = \max\left\{0, v_i\left(t+\frac{2}{3}\right) - 1\right\};$$

(4) Vehicle motion: $x_i(t+1) = x_i(t) + v_i(t+1)$,

where $d_i(t)$ is defined to be the number of empty sites in front of the ith vehicle at time t, and $v_i(t)$ is the speed of the ith vehicle at time t. $v_{\max}$ is the maximal velocity of a car. In their simulations, the road is divided into cells of length 6.5 m, i.e. the normal space headway of two successive cars in real traffic. Each cell can either be empty or occupied by just one vehicle at a certain time. Meanwhile, the time is divided into time steps of one second. Thus, a velocity of n means $n \times 6.5$ m/s.

2.2. *Asymmetrical two-route scenario with bottleneck*

In the previous research, they adopted the symmetric two-route model.[12−14,16] That is too far from the reality. In this paper, we assume that the lengths of two routes are unequal and there exists a speed limit bottleneck on one route (see Fig. 1). At every time step, a new vehicle is generated at the entrance of two-route system in probability, and then will choose one route with the feedback information.[18] Here, the definition of the arrival probability of vehicles at entrance (V_p) is the probability that vehicles arrive at entrance at each time step. For example, if $V_p = 0.7$, that means the probability of the vehicles arriving at entrance at each time step is 0.7. If a vehicle enters one of two routes, the motion of it will follow the dynamics of the VDR mechanism.

In the former work, the vehicle will be deleted if it cannot enter the route for congestion. That is obviously far from reality. In this paper, we revise this missing as follows: if a vehicle cannot enter the route, it will wait for entering at next time step.

Compared with the two-route system with one entrance and two exits, we adopt the system with one entrance and one exit (see Fig. 1). The rules at the exit of the
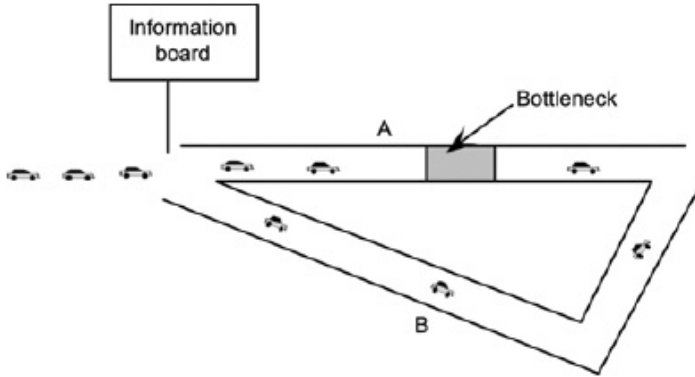
Fig. 1.   Sketch of asymmetrical two-route include bottleneck scenario.

two-route system are as follows:

(a) At the end of two routes, the car that is nearer to the exit goes first.
(b) If the cars at the end of two routes have the same distance to the exit, the one which drives faster goes out first.
(c) If the cars at the end of two routes have the same distance to the exit and have the same speed, then the cars go out randomly.

2.3. *Flux and vehicle types*

The flux is defined as follows:

$$F = V_{\text{mean}}\rho = V_{\text{mean}}\frac{N}{L}, \tag{1}$$

where L is the length of one route, N denotes the vehicle number on each road, and V_{mean} represents the mean velocity of all vehicles on each route.

There are two types of drivers that are introduced: the static type and the dynamic type. Static drivers ignore the information displayed on the board, and select one route at random. Dynamic drivers accept the information. Suppose the ratio of dynamic and static drivers is S_{dyn} and $1 - S_{\text{dyn}}$, respectively.

2.4. *Information feedback strategies*

TTFS: At the beginning, both routes are empty and the information of travel time on the board is set to be the same. Each driver will record the time when he enters one of the routes. Once a vehicle leaves the two-route system, it will transmit its travel time on the board and at that time a new dynamic driver at entrance will choose the road with shorter time.

MVFS: At every time step, each vehicle on the routes transmits its velocity to the traffic control center which will deal with the information and display the mean

velocity of vehicles on each route on the board. Road users at entrance will choose one road with larger mean velocity.

CCFS: Every time step, each vehicle transmits its signal to satellite, then the navigation system (GPS) will handle the information and calculate the position of each vehicle which will be transmitted to the traffic control center. The work of the traffic control center is to compute the congestion coefficient of each road and display it on the board. Road users at the entrance will choose one road with smaller congestion coefficient. The congestion coefficient is defined as:

$$C = \sum_{i=1}^{m} n_i^w,\tag{2}$$

where n_i stands for vehicle number of the ith congestion cluster in which cars are close to each other without a gap between any two of them. Every cluster is evaluated by a weight w, here $w = 2$.[14]

WCCFS: Every time step, the traffic control center will receive data from the navigation system (GPS) like CCFS, and the work of the center is to compute the congestion coefficient of each road with a reasonable weighted function and display it on the board. Road users at the entrance will choose one road with smaller weighted congestion coefficient. The weighted congestion coefficient is defined as:

$$C_w = \sum_{i=1}^{m} \left(k \times \frac{n_{ij}}{2000} + 2.0 \right) \times n_i^w,\tag{3}$$

where n_i stands for vehicle number of the ith congestion cluster; n_{ij} stands for the median position of the ith congestion cluster. Every cluster is evaluated by a weight w, here $w = 2^{16}$. k is a factor to be determined. This factor affects traffic flux. According to the simulation results, we found out that for new route model, average flux of two-route reaches maximum value at $k = -2.0$.

PFFS: Every time step, the transportation control center will deal with the data and then calculate the congestion coefficient on each route according to a function. We need to calculate the normalized congestion coefficient (C_p/L) because two routes' lengths are not equal. Then the information will be displayed on the board and the drivers will choose the road with the smaller congestion coefficient. The definition of the function is:

$$C_p = \begin{cases} \displaystyle\sum_{i=1}^{m} \left(k_a \times \frac{n_{ij}}{L} + a \right) \times n_i^w, & \text{if } n_{ij} \in (L_1, L_1 + \triangle L), \\ \displaystyle\sum_{i=1}^{m} \left(k_b \times \frac{n_{ij}}{L} + b \right) \times n_i^w, & \text{others}, \end{cases}\tag{4}$$

where $w = 2$, L is the length of the road. a and b are factors to be determined. The bottleneck is located between $x = L_1$ and $x = L_1 + \triangle L$. The definition of n_i and n_{ij} are the same as WCCFS. We found the system capacity depends on the parameter (k_a, k_b). We will discuss it in the next section in detail.

3. Simulation Results

We set the length of the shorter road $L_A = 5000$ and the longer road $L_B = 8000$. Traffic bottleneck appears on short road and is located between $x = L_1$ and $x = L_1 + \triangle L$. We set $L_1 = 2600$ and $\triangle L = 600$. In the bottleneck region, maximum speed of vehicles is set $M = 1$, otherwise $M = 3$. Two constants of C_p are set $a = 3$ and $b = 2$, respectively. The ratio of dynamic drivers is $S_{\text{dyn}} = 0.5$. All simulation results shown here are obtained by $200\,000$ iterations excluding the initial $100\,000$ time steps.

Firstly, we study the influence of k_a and k_b when PFFS is adopted. According to results of simulation, we find that the flux is greatest when $k_a = -3.9$ and $k_b = -2.1$. So we will use $k_a = -3.9$ and $k_b = -2.1$ in the following simulation.

In order to compare with PFFS, we need improved TTFS, MVFS and CCFS because these strategies will be invalid when they are used in the asymmetrical two-route system.

(1) ITTFS: The feedback information is L/t, where L is route length and t is travel time in TTFS.
(2) IMVFS: The feedback information is L/v, where L is route length and v is average velocity in MVFS.
(3) ICCFS: The feedback information is C/L, where L is route length and C is congestion coefficient in CCFS.

The researchers specified that a vehicle would arrive at entrance at every time step in the previous study.[12−14,16] In other words, the arrival probability of vehicles at entrance (V_p) is 1. Figure 2 shows the dependence of average flux (F_{ave}) on arrival probability of vehicles at entrance of the traffic system when four different feedback strategies are adopted. From Fig. 2(a), we find that the curves of four strategies are almost coincided when $V_p < 0.8$. Moreover, the average flux and the arrival probability of vehicles show the linear relationship with the slope of 0.5, i.e.

$$F_{\text{ave}} = 0.5 * V_p. \tag{5}$$

Due to the total flux (F_z) of two-route system is twice as large as F_{ave}, thus we obtain that

$$F_z = V_p, \tag{6}$$

when $V_p < 0.8$. That means the vehicle can enter the two-route system as soon as it arrives at entrance without waiting. It is difficult to observe the change of curves as $V_p > 0.8$ in Fig. 2(a). However, we can see from Fig. 2(b), which is the magnified
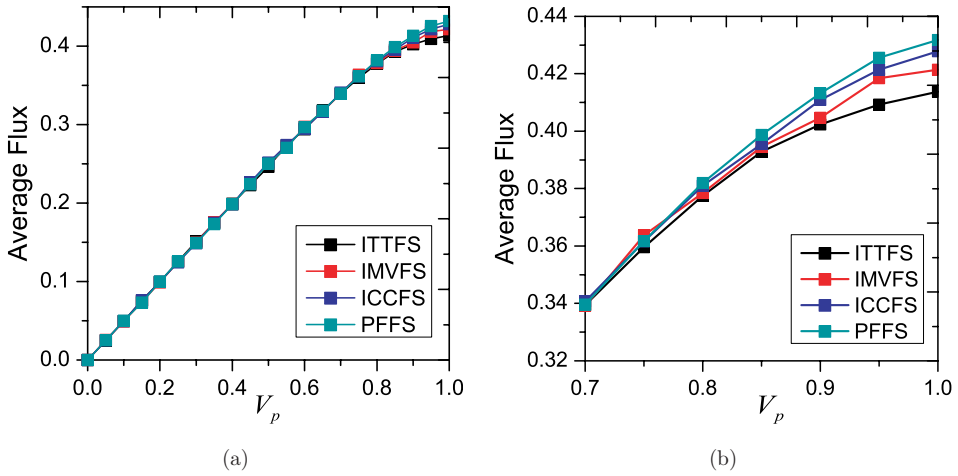
Fig. 2. (Color online) (a) Average flux versus expected arrival rate (V_p) at the entrance of the traffic system. (b) To amplify figure of (a).

version of Fig. 2(a), that the curves change flatten and the slopes of these four curves decrease with the enhancement of V_p. This is caused by the fact that vehicles cannot enter the route immediately after they arrive at entrance when $V_p > 0.8$, some of them must be waiting. In this case, we find that for an arbitrary value of V_p, the average flux of PFFS is the greatest of the four strategies. So we can conclude that PFFS is better than the other three strategies. In daily life, the arrival probability of vehicles vary with time. For instance, the value of V_p will increase during time periods of 7:00−8:00 a.m. and 5:00−6:00 p.m. Because many people go to work and come back home during that interval, so the vehicles on the route will increase. At the same time, the traffic jams are more easily occurred than any other time, the advantages of intelligent traffic systems will take effect. Therefore we set $V_p = 1$ in the simulation. At exit, vehicles will leave the route if they satisfied the rules of one-exit. In other words, the extraction probability of vehicles at exit is 1.

Figure 3 shows the changing of vehicle density according to time when adopting four different strategies. We can see that no matter which strategy we take, the density of route A is larger than the density of route B. This is because there is a speed limit bottleneck on the route A. When a car enters into the bottleneck, the maximum of speed decrease from $M = 3$ to $M = 1$, which can be more easily lead to the logjam on route A than route B. We found that the oscillatory of the density by using PFFS is smaller than ICCFS and IMVFS. Meanwhile, the values of two densities of two routes are close. Thus the balance of the density of PFFS is better than the other three strategies.

Figure 4 shows the changing of speed according to time when adopting the four strategies. We find that speed in route B is higher than that in route A by using all strategies. It can be understood as follows. Firstly, the bottleneck in route A and the
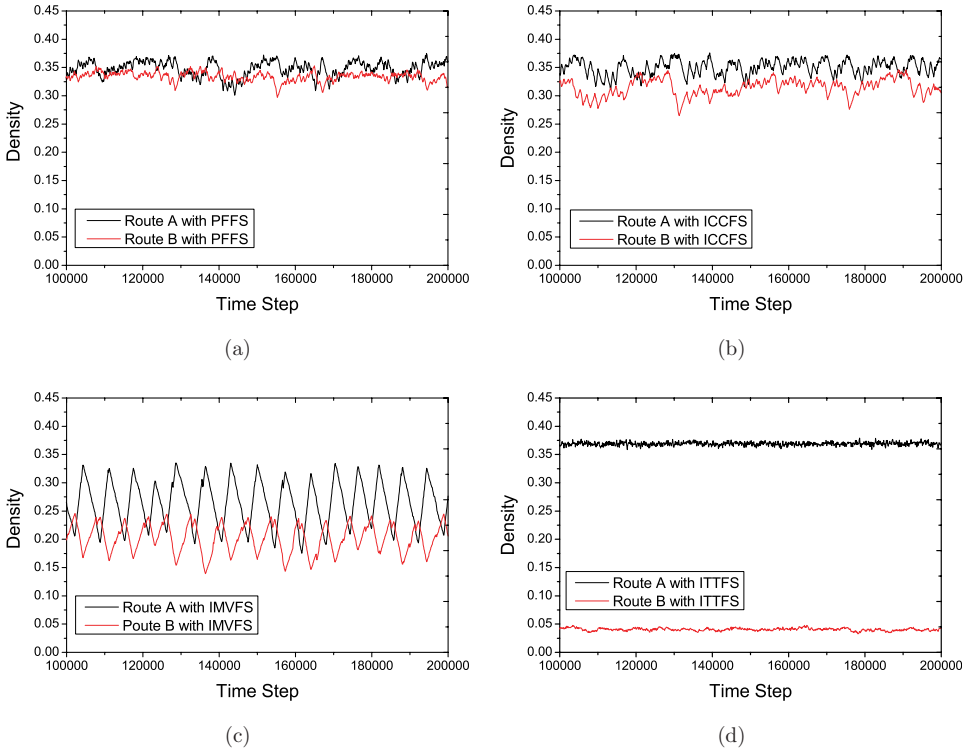
Fig. 3.   (Color online) Density of each route with (a) PFFS, (b) ICCFS, (c) IMVFS, (d) ITTFS. The parameters are $L_A = 5000, L_B = 8000, p_0 = 0.25, p_d = 0.01, S_{\text{dyn}} = 0.5$.

velocity decreases near to 1. Secondly, there is only one exit in the two-route system. According to the rule at exit (b), the vehicle on route B can drive out more easily. So it will easily generate the logjam on route A. Compared with IMVFS and ICCFS, the speed is more stable when PFFS is adopted, that is because IMVFS and ICCFS
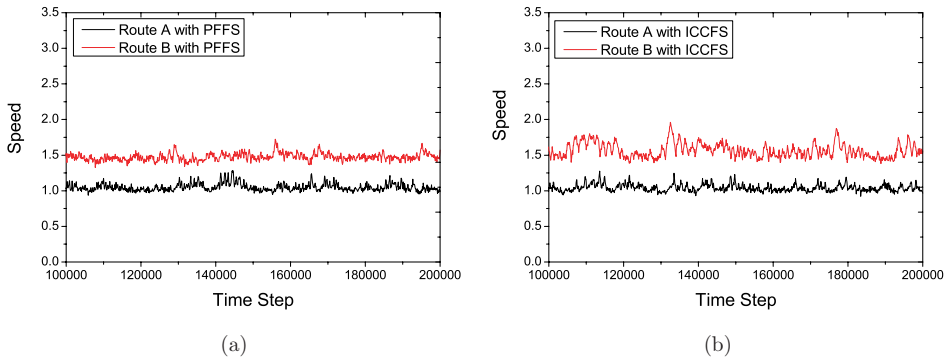
Fig. 4.   (Color online) Average speed of each route with (a) PFFS, (b) ICCFS, (c) IMVFS, (d) ITTFS. The parameters are set the same as in Fig. 3.
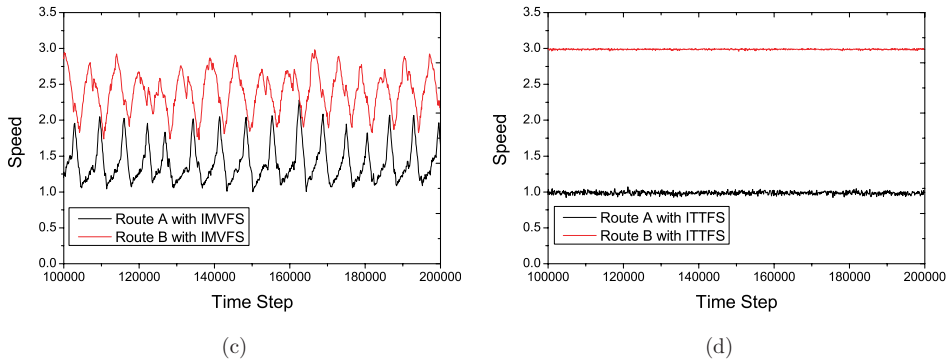
(c)   (d)

Fig. 4.   (*Continued*)

cannot describe the influence of different locations on the transportation, while it is the advantage of PFFS. When adopting PFFS, it can make the value of congestion coefficient at the end of the route smaller than before. Meanwhile, it can also make the value of congestion clusters in traffic bottleneck larger than others. Because the negative effect made by vehicles in the entrance or the bottleneck is larger than other places. Although ITTFS is superior on the stability of speed, it is not better than PFFS on the balance of speed in two routes.

Figure 5 shows the changing of flux according to time when adopting the four strategies. Its trend is almost same with Fig. 4. The only difference is that the flux is greater in route A when ITTFS is adopted. That is because the definition of flux is density multiplied by average speed. Although the value of density in route A is much greater than that in route B and the speed in route A is smaller than that in route B, the flux of route A is greater which is the product of them.
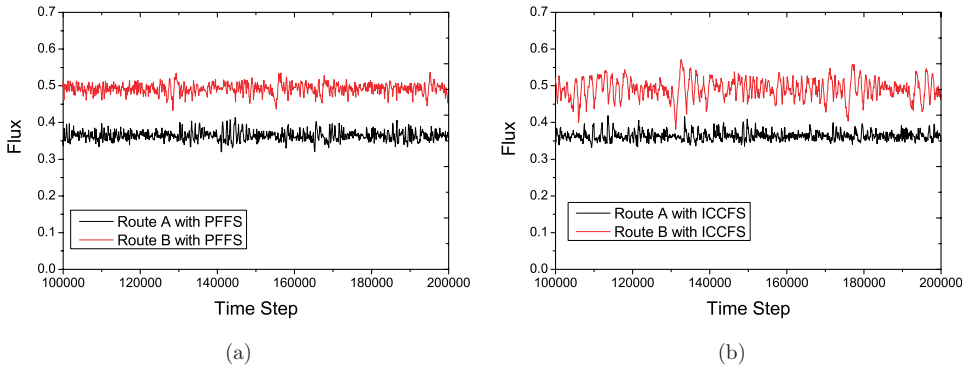

(a)   (b)

Fig. 5.   (Color online) Flux of each route with (a) PFFS, (b) ICCFS, (c) IMVFS, (d) ITTFS. The parameters are set the same as in Fig. 3.

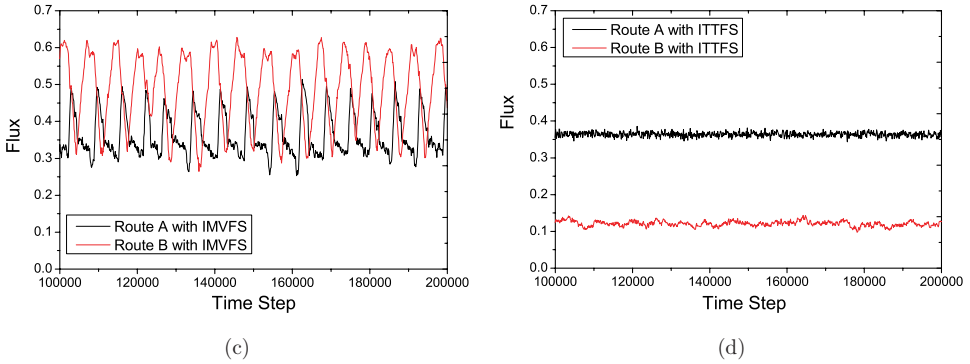(c)   (d)

Fig. 5.   (*Continued*)

As stated previously, the performance by adopting PFFS is remarkably improved, not only in the balance density of two routes, but also in the stability of flux and speed.

In previous works, NS mechanism is adopted after vehicles enter routes (see Fig. 6). Compared with VDR mechanism, adopting NS mechanism brings greater oscillatory in flux and decreases the value of flux.

Figure 7 shows how the average flux changes along with the ratio of dynamic travelers when ITTFS, IMVFS, ICCFS, WCCFS and PFFS are adopted. Maybe someone will ask why we do not use weighted congestion coefficient feedback strategy (WCCFS). We also study the work by using the WCCFS (C_W/L), and the simulation results show that the variation of flux, density and speed are almost consistent with that by using PFFS. However, from Fig. 7 we can see that the value
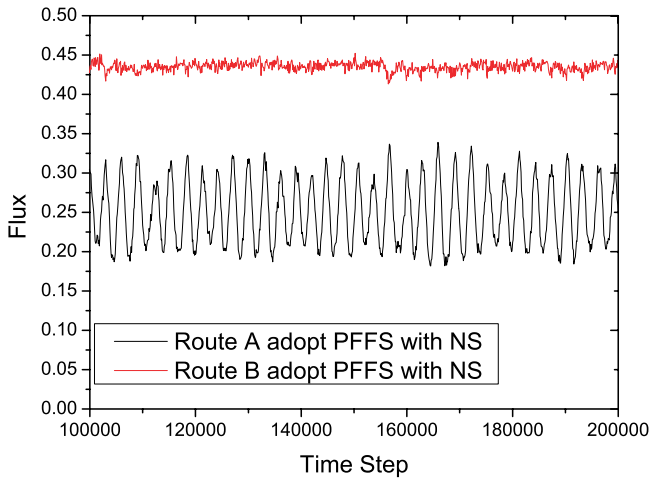

Fig. 6.   (Color online) Flux of each route adopt PFFS with NS model.
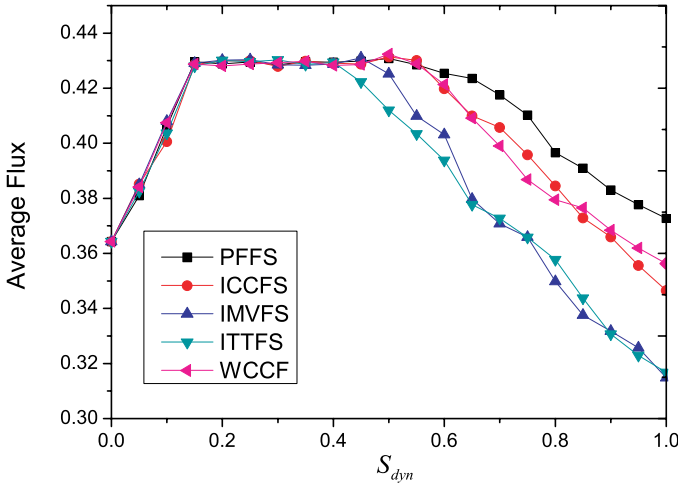
Fig. 7.   (Color online) Average flux by performing different strategy versus S_{dyn}.

of average flux by using PFFS is larger than that adopting WCCFS when $S_{\mathrm{dyn}} \geq 0.5$. Among these five feedback strategies shown in Fig. 7, only the flux adopting PFFS $(S_{\mathrm{dyn}} = 1.0)$ is higher than the flux of vehicle entering the system randomly $(S_{\mathrm{dyn}} = 0)$. This indicates that if we adopt an inappropriate information feedback, it will make the road condition even worse than the situation with vehicle entering the system randomly. The new strategy is proved to be the best one as it can get a higher flux not only than the random access $(S_{\mathrm{dyn}} = 0)$ but also than ITTFS, IMVFS, ICCFS and WCCFS at each S_{dyn} value when $S_{\mathrm{dyn}} > 0.5$. So we take PFFS as the optimal strategy in the two-route scenario with a speed limit bottleneck.

4. Conclusion

In this paper, we put forward a new strategy named piecewise function feedback strategy. We applied this new feedback strategy together with four previously proposed feedback strategies on the asymmetry two-route model with a speed limit bottleneck to get the simulation results. We obtained the dependence of vehicle number, speed and flux on time step, as well as the relationship between the average flux and the ratio of dynamic drivers. The results show that, compared with other strategies, PFFS can improve the condition of transportation effectively, not only having good stability and balance in flux, velocity and density, but also improving the average flux on two routes. The highlight of this paper is that a piecewise function of congestion coefficient is proposed. This function can give different weight value when congestion clusters locate at different locations and even if in the traffic bottleneck.

With the development of modern technology, it is not that difficult to put PFFS into practice. The information of every vehicle can be gathered by the GPS.

Then PFFS can come true through computational simulation by acting the weight value on each congestion cluster on the basis of CCFS. Taking the cost and traffic capability into consideration, we strongly believe that this strategy is quite applicable.

Acknowledgments

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