Harmonic Generation Under Small Signal Conditions in a Traveling Wave Tube

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Abstract: *In a klystron, charge overtaking of electrons leads to an infinity of AC current. The harmonic content therein can be calculated exactly, with or without space charge effects. This paper extends the klystron theory to a traveling wave tube (TWT). We assume that the electron motion is described by linear theory. The crowding of these linear orbits may lead to harmonic generation, as in a klystron. We calculate the buildup of harmonic content as a function of distance from the input, and compare these analytic results with the CHRISTINE code. Reasonable agreement was found. A dimensionless "bunching parameter" for TWT,* $X=\sqrt{2P_{in}/(P_bC)}$*, is identified, which characterizes the harmonic content in the AC current, where* P_{in} *is the power of the input signal,* P_b *is the DC beam power, and C is Pierce's gain parameter.*

Keywords: TWT, harmonic generation, nonlinearity.

In a traveling wave tube (TWT), it is recognized that the linear theory of Pierce provides an adequate description of the electron-circuit interaction over approximately 85 percent of the tube length, even when the TWT is driven to saturation. Because Pierce's theory is in the linear (small signal) regime, we are not aware of any analytic formulation that calculates the harmonic content in the AC current that would buildup over this 85 percent of tube length. We recently provided such a formulation [1].

The generation of harmonics in the small signal regime seems contradictory at first sight. Our experience with klystron theory, however, demonstrates that harmonic generation does indeed occur in the small signal regime, and that the amplitudes and phases of the harmonic currents can be calculated accurately [2]. For a TWT, we assume that the electron orbit can be described by Pierce's classical 3-wave theory of TWT. The crowding of these linear orbits may lead to harmonic generation, as in a klystron. Below, we show that this *kinematic* buildup of harmonic current is quite significant, even when the TWT operates in the small signal regime.

Consider a nonrelativistic, mono-energetic electron beam whose unperturbed velocity is v_0. It is subjected to an AC electric field, $E_{10}e^{j\omega_0 t}$, at the input (z = 0). An electron that arrives at the input at time $t = t_0$ will reach a downstream position (z) at time t, $z = v_0(t-t_0)+z_1(t,t_0)$, where $z_1(t,t_0)=\mathrm{Re}[Z_1(t,t_0)]$ is the displacement from the unperturbed orbit, given by

$$Z_1(t,t_0) = -\frac{eE_{10}}{m\omega_0^2C^2}e^{j\omega_0 t_0}\times \left[\alpha_1 e^{C\omega_0\delta_1(t-t_0)}+\alpha_2 e^{C\omega_0\delta_2(t-t_0)}+\alpha_3 e^{C\omega_0\delta_3(t-t_0)}\right], \quad (1)$$

according to Pierce's linear 3-wave theory. In Eq. (1), δ_1, δ_2, δ_3 are the three complex roots to the Pierce dispersion relation, (in Pierce's notation)

$$(\delta^2+4QC)(\delta+jb+d) = -j(1+Cb)^2,$$

where b, C, d, and QC are the usual Pierce parameters and α_1, α_2, and α_3 are the amplitudes of the three forward waves in Pierce's theory. Thus launching loss is included, and is displayed in Eq. (1) in Lagrangian form [1].

The linear orbital equation, Eq. (1), allows the calculation of the electron's arrival time (t) in terms of its departure time (t_0) at a downstream position, z = L. At z = L, the total current is $I(L,t) = I_0 + \mathrm{Re}\sum_{n=1}^{\infty} I_n e^{jn\omega_0 t}$, where

$$I_n = \frac{I_0}{\pi}\int_0^{2\pi} d(\omega_0 t_0) e^{-jn\omega_0 t_0 - jn\omega_0(t-t_0)} \quad . \quad (2)$$

Equation (2) is valid even when charge overtaking occurs [2]. From Eq. (1), a dimensionless bunching parameter X, measuring $\omega_0 Z_1/v_0$, may be identified [1],

$$X = \frac{eE_{10}}{m\omega_0 v_0 C^2} = \frac{1}{C^2}\left(\frac{v_w}{v_0}\right) = \sqrt{\frac{2}{C}\left(\frac{P_{in}}{P_b}\right)}, \quad (3)$$

where $v_w = eE_{10}/m\omega_0$ is the characteristic electron "wiggling velocity" associated with the AC electric field at the input, P_{in} is the power of the input signal, and P_b is the DC beam power.

As an example, we consider a C-band helix TWT with operating parameters $\omega_0 = 2\pi\times 4.5GHz$, V_b = 2.776 kV, I_0 =

0.17 A, C = 0.1194, K = 111.2 ohm, $v_0 = 5.93\times10^7 m/s$, P_b = V_b I_0 = 417.9 W, and $I_0 / V_b^{3/2}$ = 1.16 micro-perveance. For an input drive of P_{in} = 54 mW, X = 0.04652, this corresponds to the input power that produces a 1 dB compression (reduction) of the gain in the case QC=0, and so the TWT may be considered to be operating in the slightly non-linear regime at z=5cm in this case. The solid lines in Fig. 1 show the evolution of the RF power for (b, QC) = (0,0), (1,0), and (0, 0.296) according to Pierce's linear theory. For comparison, the CHRISTINE simulation results are shown by the dotted lines. Excellent agreement is noted. Since CHRISTINE is a nonlinear code, Fig. 1 suggests that the linear theory applies up to $z = L = 4.5$ cm. The evolution of the harmonic contents in the beam current for the cases of (b, QC) = (1,0) is shown in Fig. 2. It is seen that the harmonic content in the AC current is quite significant even when the TWT operates in the linear regime. These harmonic AC currents are due only to kinematic bunching (i.e., orbital crowding) in the electron orbits, and is predicted reasonably well by the analytic theory for z up to ~4 cm. Figure 3 shows the harmonic content at $z = 4$ cm, which is quite sizeable even in this linear regime. The harmonic current can be fairly predicted by our analytic theory.

The harmonic currents shown in Figs. 2 and 3 do not have any intrinsic gain mechanism, because they are due only to *kinematic* bunching of the linearized orbit at the fundamental frequency, ω_0. This is not necessarily true in a wideband TWT, where the second harmonic ($2\omega_0$) may also be within the amplification band. Should this be the case, the second harmonic current calculated here may then be considered as a seed for power generation at the second harmonic (but without any $2\omega_0$ input). Note that the amplitude and phase of this $2\omega_0$ output are completely controlled by the input signal, which is at the fundamental frequency ω_0. Results on these studies will be reported.

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References

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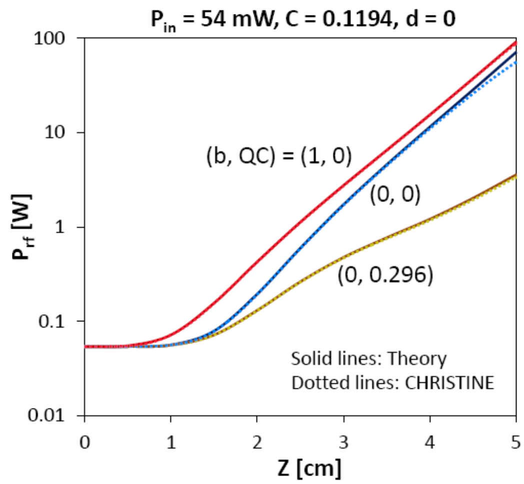

Fig. 1. Evolution of RF power on the circuit wave for various combinations of (b, QC).

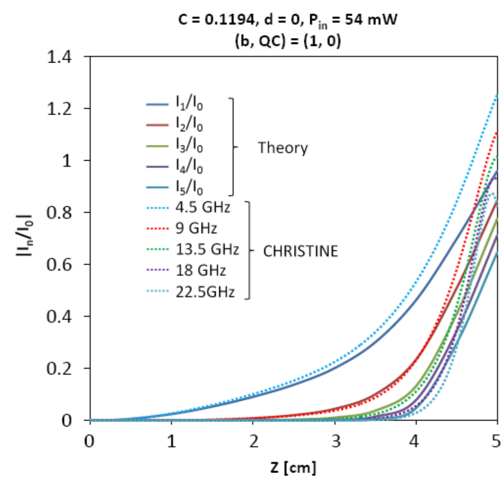

Fig. 2. Harmonic content as a function of z, in linear scale, for (b, QC) = (1, 0).

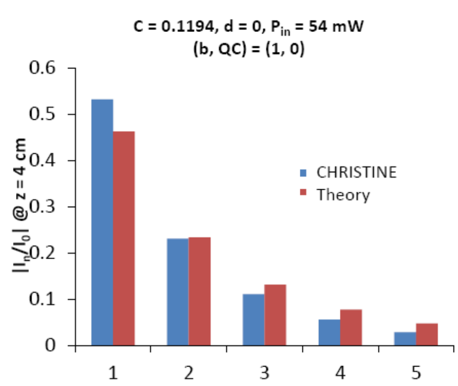

Fig. 3. Harmonic content at z = 4cm for (b, QC) = (1, 0).